Introduction to Precise Point Positioning

Prof. Yang Gao
Department of Geomatics Engineering
The University of Calgary

ygao@ucalgary.ca
http://people.ucalgary.ca/~point/

Prepared in Dec/2004 and updated in July/2012

Copyright Disclaimer

Acknowledgements

The author would like to acknowledge the assistance of his graduate students during the preparation of this document

CONTENTS

Section 1

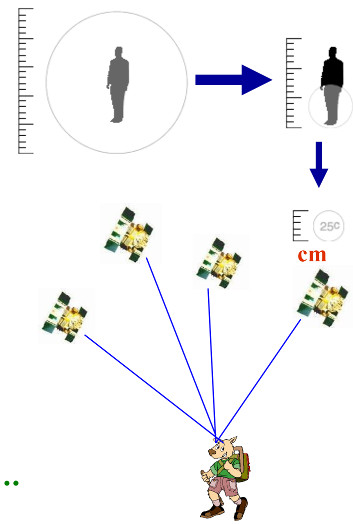
Concept of Precise Point Positioning (PPP)

- GPS Positioning Method Overview
- What is PPP?
- Why PPP?
- Major Issues and Challenges

GPS Positioning Method Overview (1/11)

Autonomous GPS (this is what GPS has been designed for!)

- Using a single GPS receiver
  - Real-time
  - Kinematic
  - Absolute positioning
  - Several meter accuracy
- Requirements
  - Line of sight view to at least 4 GPS satellites from the receiver
- Limitations
  - Positioning is subject to the effect of all GPS error sources
  - Low accuracy

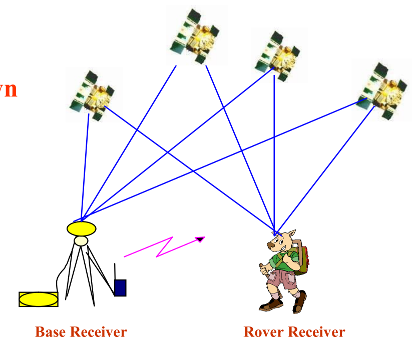


…people demand more and more ….

GPS Positioning Method Overview (2/11)

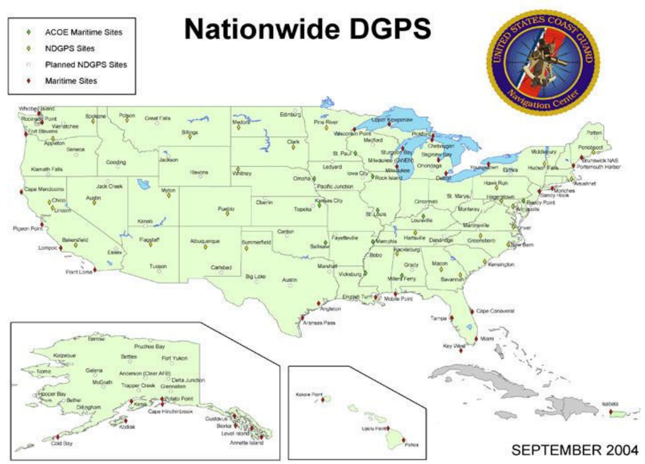
Differential GPS (major errors are spatially correlated)

- Code based Differential GPS
  - Corrections (position or measurement corrections) are calculated at a base station
  - Correction are sent to user (rover) stations
  - Real-time
  - Kinematic
  - Metre-level accuracy
- Requirements
  - A base station with precisely known coordinates
  - User station be within the vicinity of base stations
- Limitations
  - Relative positioning
  - May increase operational cost



GPS Positioning Method Overview (3/11)

- Broadcast station generated code measurement corrections to users using beacon stations

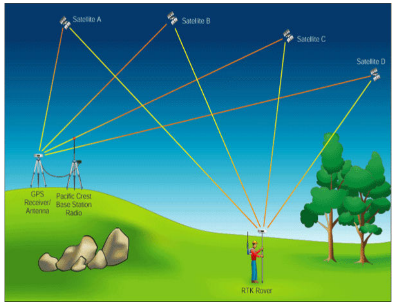


US Coast Guard Maritime Differential GPS Service

(Source: http://www.navcen.uscg.gov/dgps/coverage/SiteLocations.htm)

GPS Positioning Method Overview (4/11)

Differential GPS



- **Carrier Phase based Differential GPS**
  - Measurements or measurement corrections are calculated at a base station and then sent to user (rover) stations
  - Real-time Kinematic (RTK)
  - Centimetre accuracy
- **Requirements**
  - Simultaneous observations and double differencing between the base and user stations
  - Very short rover-base station separation typically < 20 km
- **Limitations**
  - Relative positioning
  - Increased operational cost and logistical complexity

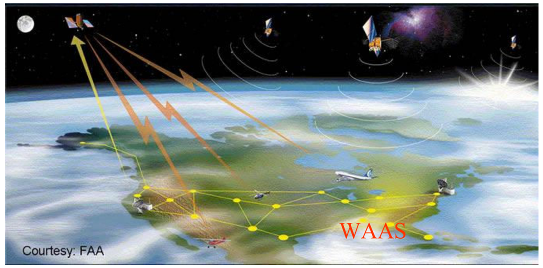
Carrier Phase based Differential Real-Time Kinematic (RTK) Positioning Technology has revolutionized the practice of geodetic positioning

GPS Positioning Method Overview (5/11)

Differential GPS (network approach):

Satellite-Based Augmentation Systems (SBAS)

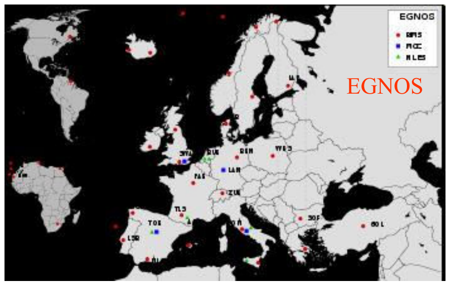
- **Code based Differential GPS**
  - State-space corrections for orbit, clock and ionosphere are calculated by the network
  - Correction are sent to user (rover) stations via communication satellites
  - Real-time
  - Kinematic
  - Metre-level accuracy
- **Requirements**
  - Most receivers are SBAS enabled
  - User station be within the network



GPS Positioning Method Overview (6/11)

Differential GPS (network approach)

- **Different SBAS Systems**
  - U.S.A - Wide Area Augmentation System (WAAS)
  - Europe - European Geostationary Navigation Overlay Service System (EGNOS)
  - Japan - MTSAT Satellite Augmentation System (MSAS)
  - China – BeiDou system (CSAS)
  - Canadian DGPS (CDGPS)
  - ……

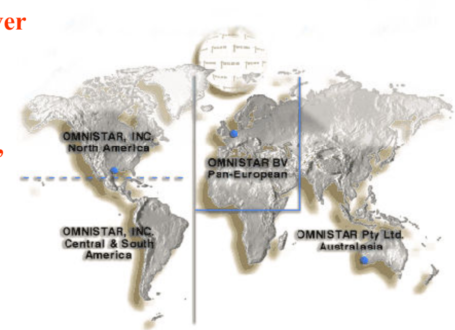


GPS Positioning Method Overview (7/11)

Differential GPS (network approach)

- **Commercial Differential Services**
  - Differential Corrections Inc (service via wireless network, regional)
  - Accupoint (service via FM subcarrier, over North America)
  - SatLoc (service via satellite, over North America)
  - LandStar (service via satellite, regional)
  - OminStar (service via satellite, worldwide)
  - NavCom (service via satellite, worldwide)
  - ……

OmniSTAR Service Regions

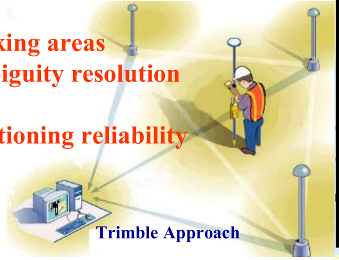


GPS Positioning Method Overview (8/11)

Differential corrections are calculated based on data from a network of reference stations

Network RTK

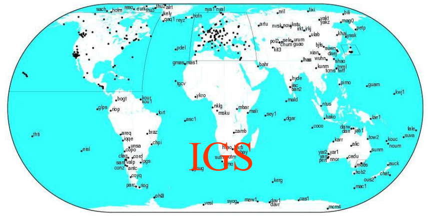
- Still double differenced data processing
- Extend baseline length (several tens of kilometres)
- Extended working areas
- Improved ambiguity resolution performance
- Improved positioning reliability

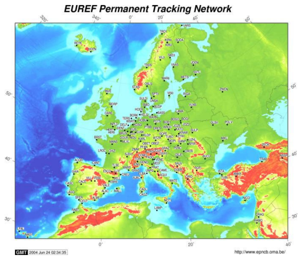


GPS Positioning Method Overview (9/11)

Significant number of permanent reference GPS stations have been deployed worldwide

New Development

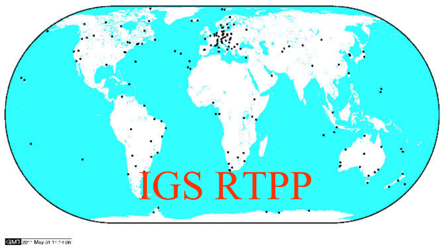


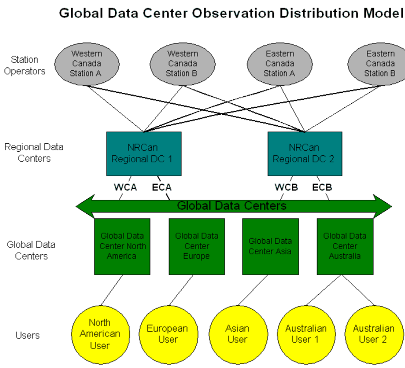


- Reference station infrastructure
  - Permanent
  - At precisely known stations
  - High quality receivers
  - 24-h continuous tracking
  - Can be used as base stations
- Station deployment
  - Local area
  - Regional area
  - Wide area
  - Global
- Limitations
  - Not always available at desired areas

GPS Positioning Method Overview (10/11)

IGS Real-Time Pilot Project (RTPP) was initialized in June 2007 (http://www.rtigs.net/index.php).

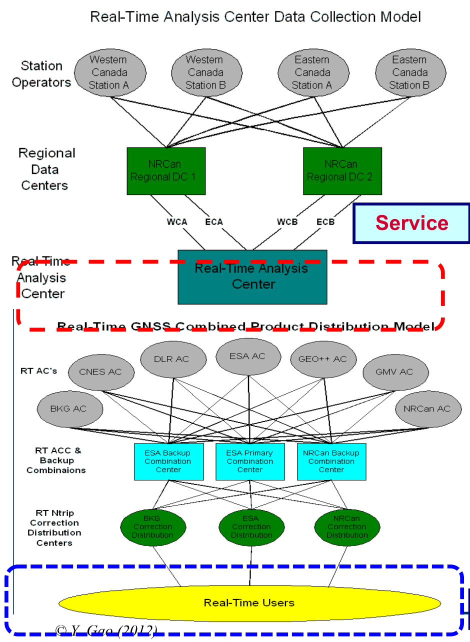




RTPP data format

	RTCM type	Description
Station/Antenna	1006 1008	Station coordinates Receiver antenna type
Observation	1004 1012	GPS dual-frequency obs GLONASS dual-frequency obs
Ephemeris	1019 1020	GPS ephemeris GLONASS ephemeris

GPS Positioning Method Overview (11/11)

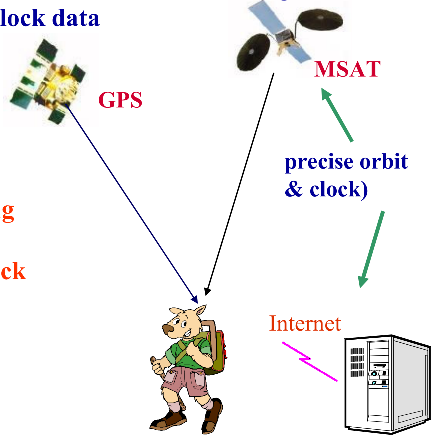


GNSS Space State Representative (SSR) was proposed in 2005. The SSR messages are described by RTCM type 1057-1068.

	RTCM type	Description
GPS SSR	1057 1058 1059 1060 1061 1062	Orbit correction Clock correction Code biases Orbit/clk correction Sat URA High-rate clock correction
GLONASS SSR	1063 1064 1065 1066 1067 1068	Orbit correction Clock correction Code biases Orbit/clk correction Sat URA High-rate clock correction

What is PPP?

- The availability of precise GPS satellite orbit and clock products has made it feasible to process un-differenced code and carrier phase observations for precise position determination.
- Precise Point Positioning is a method based on the processing of (usually) un-differenced code and carrier phase observations from a single GPS receiver and precise GPS orbit and clock data



Features

- Un-differenced data processing
- Absolute positioning
- Requires precise orbit and clock
- cm ~ dm positioning accuracy
- Point RTK

Why PPP?

- Single receiver operation
- Precise positioning without a need for base stations
- Absolute positioning method (global)
- Direct Access to Global Reference Frame
- Accuracy comparable to differential approach
- Point RTK possible with real-time precise orbit and clock products
- Reduced operational cost (equipment and labor)
- Simplified logistics in the field
- Greater operational flexibility
- Increased capability beyond positioning
- Open up many new applications
- ……

Major Issues and Challenges

- GPS Error and Mitigation
  - Positioning is subject to the effect of all GPS error sources
  - Require various error mitigation and modeling (Satellite-related errors, Propagation-related errors, Receiver-related errors)
- Precise Orbit and Clock Products
  - Post-mission precise GPS orbit and clock products
  - Real-Time precise orbit and clock products
  - Availability and distribution
- PPP Positioning Methods
  - Observation models
  - Stochastic models
  - Estimation methods
  - Fast float solution convergence
  - Integer ambiguity resolution
  - Dual-frequency PPP
  - Single-frequency PPP

Section 2

Measurement Error, Effect and Mitigation

- Measurements (Un-differenced)
- Measurement Errors and Effects
- Mitigation Methods
- Measurement Corrections
  - ✓ Satellite-related corrections
  - ✓ Propagation-related corrections
  - ✓ Receiver-related corrections

Measurements (Un-differenced)

Code and phase measurement

$$
\begin{cases}
P_1 = \rho + d_{orb} + c(dt^r - dt^s) + T + I_1 + b^r_{P1} - b^s_{P1} + \varepsilon_{P1} \\
P_2 = \rho + d_{orb} + c(dt^r - dt^s) + T + I_2 + b^r_{P2} - b^s_{P2} + \varepsilon_{P2} \\
L_1 = \rho + d_{orb} + c(dt^r - dt^s) + T - I_1 - \lambda_1 N_1 + b^r_{L1} - b^s_{L1} + \varepsilon_{L1} \\
L_2 = \rho + d_{orb} + c(dt^r - dt^s) + T - I_2 - \lambda_2 N_2 + b^r_{L2} - b^s_{L2} + \varepsilon_{L2}
\end{cases}
$$

where i = 1, 2

P_i is the raw code measurement (m);
L_i is the raw phase measurement (m);
ρ is the geometric distance as a function of rcv and sat coordinates (m);
d_{orb} is the satellite orbital error (m);
c is the speed of light in vacuum (m/s);
dt^r is the common receiver clock (s);
dt^s is the common satellite clock (s);
T is the tropospheric delay (m);
I_i is the first-order ionospheric delay on frequency i (m);
λ_i is the wavelength of carrier phase on frequency i (m);
N_i is the integer ambiguity on frequency i (cycle);
b^r_* is the observable-dependent receiver bias (m);
b^s_* is the observable-dependent satellite bias (m);
ε_{Pi} is the code remaining error such as multipath and noise (m);
ε_{Li} is the phase remaining error such as multipath and noise (m).

Measurement Error and Effects

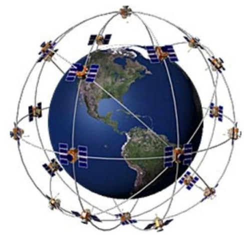
Error/effect type	Error/effect term
Satellite-related	Satellite orbit error
	Satellite clock error
	Relativistic effect
	Satellite antenna phase center offset
	Phase wind-up
Propagation-related	Troposphere
	Ionosphere
Receiver-related	Receiver antenna phase center offset
	Sagnac effect
	Receiver clock error
	Multipath
	Solid earth tides
	Polar tides
	Ocean loading
	Earth rotation parameters

Mitigation Methods

	Error term	Mitigation method
Satellite-related	GPS orbit and clock	- Satellite broadcast message - IGS products
	Relativistic effect	Model correction
	Satellite antenna phase center correction	IGS ANTEX
	Phase wind-up	Model correction
Propagation-related	Troposphere	- ZHD modeling - ZWD estimation
	Ionosphere	- Linear combination - External ionosphere products
Receiver-related	Receiver antenna phase center correction	IGS ANTEX
	Sagnac effect	Model correction
	Receiver clock error	Estimation
	Solid earth tides	Model correction
	Polar tides	Model correction
	Ocean loading	Model correction
	Earth rotation parameters	Model correction

Satellite-related correction - GPS orbit and clock

- **Satellite orbit error**
  - ✓ Broadcast orbit errors: a few meters after SA was turned off
  - ✓ Spatially correlated between receivers with common view to it



- **Satellite clock error**
  - ✓ Very accurate (Cesium) atomic clocks
  - ✓ Clock errors (offset and drift) are calculated and included in ephemeris
  - ✓ Clock errors (after correction using the broadcast clock parameters): ~ 10 ns level after SA was turned off
  - ✓ Same to all receivers

Satellite-related correction - GPS orbit and clock

GPS satellite orbit and clock product summary
(Available at http://igscb.jpl.nasa.gov/components/prods.html on 2012-04-30)

		Accuracy	Latency	Updates	Sample Interval
Broadcast	Orbit	~ 100 cm	real-time	--	2 hours
	Clock	~ 3 ns RMS ~ 2.5 ns STD			
Ultra-Rapid (predicted half)	Orbit	~ 5 cm	real-time	at 03, 09, 15, 21 UTC	15 min
	Clock	~ 3 ns RMS ~ 1.5 ns STD			
Ultra-Rapid (observed half)	Orbit	~ 3 cm	3 ~ 9 hours	at 03, 09, 15, 21 UTC	15 min
	Clock	~ 150 ps RMS ~ 50 ps STD			
Rapid	Orbit	~ 2.5 cm	17 ~ 41 hours	at 17 UTC daily	15 min
	Clock	~ 75 ps RMS ~ 25 ps STD			5 min
Final	Orbit	~ 2.5 cm	12 ~ 18 days	Every Thursday	15 min
	Clock	~ 75 ps RMS ~ 20 ps STD			30s

Satellite-related correction - Relativistic effect

✓ The relativistic effect is valid for two objects at different motions and gravity potentials. For example, the GPS satellites travel at a speed of about 4 km/s and an altitude of about 20200 km with respect to the user receiver located on the earth surface.

✓ For GPS satellites, the frequency drift amounts to about $4.5*10^{-10}$, which consists of a constant offset of about $4.6*10^{-10}$ and a periodical variation.

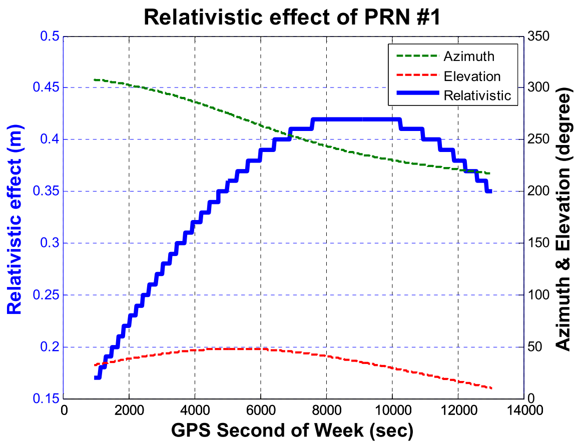
✓ The constant frequency drift offset can been compensated through adjusting the atomic clock frequency by the satellite clock manufacturer before launching the satellite.

✓ The periodical variation is identified because of the circular satellite orbit assumption. The effect of the orbit eccentricity results in the relativistic effect as

$$d_{rel} = -\frac{2v_{sat} * x_{sat}}{c^2}$$

- v_{sat} the satellite velocity
- x_{sat} the satellite position
- c the speed of light in vacuum

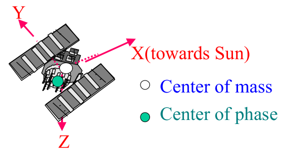
Satellite-related correction – Relativistic effect



PRN #1 using data collected at PRDS on 2012/01/01

Satellite-related correction – Satellite antenna phase center offset

Revolution of satellite antenna phase center correction models:



1. Relative satellite antenna correction before 2006/11/05
- Block-specific PCO in the Z- and X- directions;
- No PCV.

2. Absolute satellite antenna correction before 2011/04/17
- Satellite-specific PCO in the Z-, Y- and X- directions;
- Nadir-dependent PCV;
- Consistent with ITRF2005.

3. Absolute satellite antenna correction since 2011/04/17
- Satellite-specific PCO in the Z-, Y- and X- directions;
- Nadir-dependent PCV;
- Consistent with IGS08;
- Available at ftp://igs.org/igscb/station/general/igs08.atx.

Satellite-related correction – Satellite antenna phase center offset

Example

- GPS satellite PRN#01 on 2012-05-05
- Nadir 7.5 degree

Procedure

First, the satellite PCOs are listed as 394.00/0.00/1650.00 mm in the NEU directions.

Second, the tabulated satellite PCVs of two grid points around the nadir angle are obtained

Third, a linear interpolation is applied to derive the nadir-dependent satellite PCV.

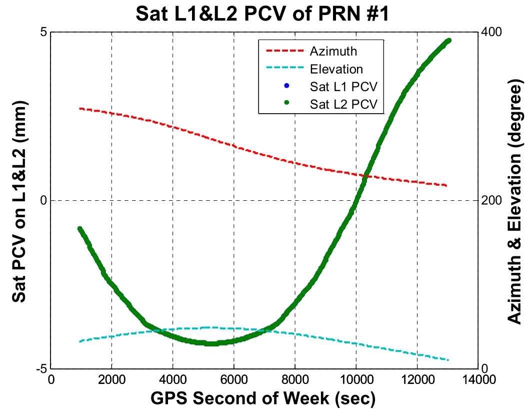
Result

Satellite antenna PCO and PCV corrections as the example

Frequency	PCO in NEU (mm)	PCV (mm)		
		Tabulated values		Interpolated value
Iono-free	*394.00/0.00/1650.00*	-3.90	-4.40	*-4.15*

Satellite-related correction – Satellite antenna phase center offset

- ✓ **Sat PCVs on L1 and L2 are the same;**
- ✓ **Consistent Sat PCO are used to calibrate sat coordinates.**



Sat L1&L2 PCV corrections of PRN #1

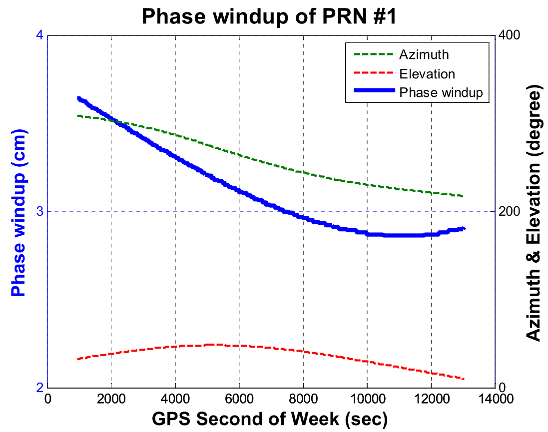
Satellite-related correction – Phase wind-up

- ✓ **The rotation of either the satellite or receiver antenna would cause the change of phase measurement up to one cycle (Kouba 2009), which becomes pretty critical to the integer phase ambiguity resolution. This effect is called phase wind-up (Wu et al 1993).**
- ✓ **The phase wind-up correction can be represented as**

$$\Delta\phi = sign(\zeta) * \cos^{-1}\left(\frac{\vec{D}' \cdot \vec{D}}{|\vec{D}'||\vec{D}|}\right)$$

$(\hat{x}, \hat{y}, \hat{z})$	local receive unit vector	$\vec{D} = \hat{x} - \hat{k}(\hat{k} \cdot \hat{x}) - \hat{k} \times \hat{y}$
$(\hat{x}', \hat{y}', \hat{z}')$	satellite body coordinate unit vector	$\vec{D}' = \hat{x}' - \hat{k}(\hat{k} \cdot \hat{x}') - \hat{k} \times \hat{y}'$
$\hat{k}$	satellite-receiver unit vector	$\zeta = \hat{k} \cdot (\vec{D}' \cdot \vec{D})$

Satellite-related correction – Phase wind-up

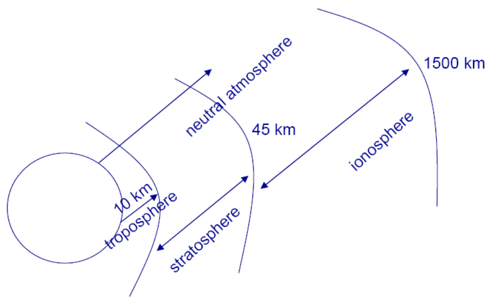


Phase wind up offset correction of PRN #1

Propagation-related correction

✓ GPS signals travelling through the earth's atmosphere experience both the distance delay/advance and bending effects. The signal bending effects are always not concerned in GPS positioning applications due to the small magnitude (<3 mm for >20 degree elevation angle) (Skone 2009).

✓ In general, the atmosphere can be divided into several layers according to the altitudes above the earth surface. Two major layers are ionosphere (45~1500 km) and troposphere (0~40 km).



(Source: Skone 2009)

Propagation-related correction – troposphere

✓ Troposphere effects include a wet delay at the lower region (0~10 km), and a hydrostatic delay at altitudes 0~45 km.

✓ Two approaches to calculate troposphere zenith delay

1. Refractivity profile – radiosonde and radio occultation;

$$d_{trop}{}^{z} = d_{hydro}{}^{z} + d_{wet}{}^{z} = 10^{-6}\int_s N_h ds + 10^{-6}\int_s N_w ds$$

2. Meteorological parameter model – GPS applications.

$$d_{hydro}{}^{z} = \frac{[(0.0022768 \pm 0.0000005)]P_0}{f_s(\phi, H)}$$

$$d_{wet}{}^{z} = \frac{0.002277(\frac{1255}{T_0} + 0.05)e_0}{f_s(\phi, H)}$$

✓ Elevation-dependent functions are required to map the zenith delay to the slant path.

$$m = \frac{1 + \frac{a}{1 + \frac{b}{1 + c}}}{\sin(e) + \frac{a}{\sin(e) + \frac{b}{\sin(e) + c}}}$$

Propagation-related correction – troposphere

✓ Troposphere zenith hydrostatic delay (ZHD) can be precisely modeled with real pressure data.

$$d_{hydro}{}^{z} = \frac{[(0.0022768 \pm 0.0000005)]P_0}{f_s(\phi, H)}$$

where: P_0 surface pressure in hPa at the user location;

$f_s(\phi, H) = 1 - 0.00266\cos(2\phi) - 0.00000028H$ function of the user latitude and height.

✓ Once the precise pressure data is available, ZHD can be calculated with 0.2 mm accuracy.

✓ Sometimes, the real meteorological data are not available. Global Pressure and Temperature (GPT) model as the current IERS and IGS standard is recommended to be consistent with IERS and IGS products. Source code available @ http://mars.hg.tuwien.ac.at/~ecmwf1/gpt.f.

Propagation-related correction – troposphere

✓ Troposphere zenith wet delay (ZWD) can be precisely modeled with real pressure data.

$$d_{wet}{}^{z} = \frac{0.002277(\frac{1255}{T_0} + 0.05)e_0}{f_s(\phi, H)}$$

where: T_0 temperature in Kelvin; e_0 water vapor pressure;

$f_s(\phi, H) = 1 - 0.00266\cos(2\phi) - 0.00000028H$ function of the user latitude and height.

✓ Unlike the ZHD modelling, ZWD is hard to be precisely modeled.

✓ In practice, ZWD is estimated in the GPS positioning applications, for example, as a random walk, a random constant within one hour.

Propagation-related correction – troposphere

✓ **Troposphere mapping functions (MFs) are based on three-term continued fractional form. The key to calculate MFs is to obtain the consistent set of coefficients (a,b,c).**

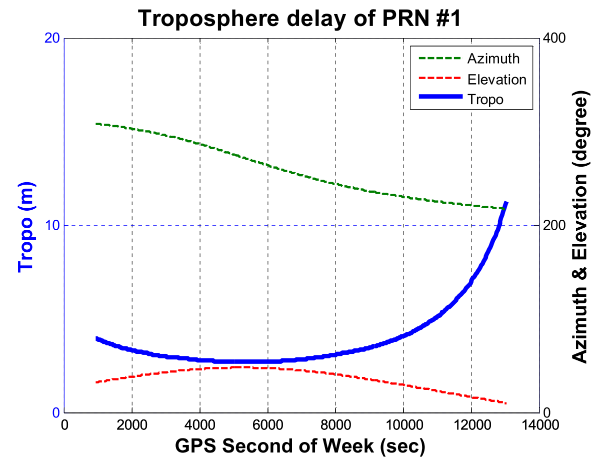
$$m = \frac{1+\dfrac{a}{1+\dfrac{b}{1+c}}}{\sin(e)+\dfrac{a}{\sin(e)+\dfrac{b}{\sin(e)+c}}}$$

✓ **Mapping function models:**

1. **Niell mapping function (NMF);**
2. **Vienna mapping function (VMF1)**
   - **latest IERS standard; latency required to retrieve coefficient file;**
3. **Global mapping function (GMF) – latest IERS standard.**
   - **latest IERS standard; no latency required; empirical coefficient set.**

Source code available @ http://mars.hg.tuwien.ac.at/~ecmwf1/.

Propagation-related correction - troposphere



Troposphere delay correction of PRN #1

Propagation-related correction – ionosphere

The ionosphere zenith delay in the form of refractive index profiles

$$d_{iono}^{\ z} = \int_s (n-1)ds$$

For the **phase** measurement

$$n_p = 1 - \frac{1}{2}X \pm \frac{1}{2}XY_L - \frac{1}{8}X^2 - \frac{1}{4}X\cdot Y^2(1+\cos^2(\theta))$$

Ionosphere delay concerning first-, second- and third-order

$$d_{iono\ \ phase}^{\ z} = -\frac{40.3}{f^2}VTEC - \frac{s_2}{f^3} - \frac{s_3}{f^4}$$

For the **code** measurement

$$n_c = n_p + f\frac{dn_p}{df}$$

Ionosphere delay concerning first-, second- and third-order

$$d_{iono\ \ code}^{\ z} = \frac{40.3}{f^2}VTEC + 2\frac{s_2}{f^3} + 3\frac{s_3}{f^4}$$

1. **Phase advance and Code delay;**
2. **First-order code and phase ionosphere effects have the same magnitude in opposite signs.**

Propagation-related correction – ionosphere

For the **phase** measurement

$$n_p = 1 - \frac{1}{2}X \pm \frac{1}{2}XY_L - \frac{1}{8}X^2 - \frac{1}{4}X\cdot Y^2(1+\cos^2(\theta))$$

where

$$X = \frac{Ne^2}{\varepsilon_0 m\omega^2} = \frac{\omega_p^2}{\omega^2}$$ → Plasma frequency

$$Y_L = \frac{eB_L}{m\omega} = \frac{\omega_L}{\omega}$$ → Gyrofrequency

$$Y_T = \frac{eB_T}{m\omega} = \frac{\omega_T}{\omega}$$ → Gyrofrequency

$$Z = \frac{\nu}{\omega}$$ → Collision frequency

Skone (2009).

Propagation-related correction – ionosphere

✓ **Magnitudes of the first-, second- and third-order ionosphere effects for GPS L1/L2 freq**

Frequency (MHz)	Technique	First-order (mm)	Second-order (mm)	Third-order (mm)
1228	GPS L1	$-8.0*10^4$	$-1.8*10^1$	$-1.1*10^0$
1575	GPS L2	$-4.8*10^4$	$-8.5*10^0$	$-3.9*10^{-1}$

✓ **The first-order ionospheric error accounts for ~ 99.9% of the total error, while the second-order error < 0.1%.**

✓ **The first-order ionospheric errors must be corrected.**

✓ **First-order ionosphere effects on the phase and code measurements (*f* is the carrier phase frequency)**

$$\begin{cases} d_{iono}{}^{z}{}_{phase} = -\dfrac{40.3}{f^2} VTEC \\ d_{iono}{}^{z}{}_{code} = \dfrac{40.3}{f^2} VTEC \end{cases}$$

Key to calculate the ionosphere effect is the vertical total electron content (VTEC).

TEC is integrated electron density along a 1 m^2 column where **1 TECU = 10^{16} el/m^2 = 27 cm on GPS L1 = 16 cm on GPS L2**

Propagation-related correction – ionosphere

✓ **IGS final IONosphere EXchange (IONEX) product can provide the best accuracy with 2~8 TEC unit (TECU) in a global 5*2.5 degree grid with ~ 11-day latency. Alternatively, the rapid IONEX product can reduce the latency to < 24 hours with 2~9 TECU accuracy.**

✓ **Example: lat/lon/hgt as (87degree, 100 degree, 450km) on 2012/01/01**

```
  2012     1     1     0     0     0                        EPOCH OF CURRENT MAP
   87.5-180.0 180.0    5.0 450.0                            LAT/LON1/LON2/DLON/H
   75   76   75   75   75   75   75   75   75   76   76   76   76   75   75   76
   76   77   76   76   75   74   72   72   70   69   68   67   66   66   65   65
   64   63   64   62   62   62   62   62   61   60   60   61   60   60   60   60
   60   60   60   61   61   61   62   62   63   64   64   66   67   67   68   69
   71   71   73   73   74   74   75   74   75
   85.0-180.0 180.0    5.0 450.0                            LAT/LON1/LON2/DLON/H
   78   79   81   83   84   85   86   86   87   87   87   86   86   85   84   83
   81   81   79   78   77   77   76   74   72   70   68   67   65   60   59   57
   56   55   54   53   53   52   52   52   52   53   54   54   56   56   57   57
   58   58   59   59   60   60   61   61   62   63   64   65   66   67   68   69
   71   72   73   73   74   75   76   77   78
```

VTEC at (87.5, 100, 450) is 63*0.1=6.3VTEC unit

VTEC at (85 , 100, 450) is 62*0.1=6.2VTEC unit

Linear interpolation

VTEC at (87 , 100, 450) is **6.28VTEC unit**

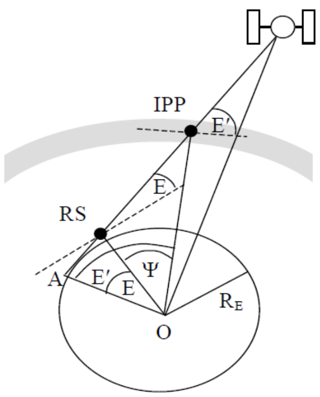
6.28 VTEC unit = 1.6956 m on GPS L1 = 1.0048 m on GPS L2

Propagation-related correction – ionosphere

✓ **Once the ionosphere zenith delay is obtained, an elevation-dependent function is required to map the zenith delay to the slant path. A spherical ionosphere shell at a latitude of 450 km is always assumed.**

$$m = \frac{1}{\sqrt{1 - \dfrac{r^2 \cos^2(E)}{(r+h)^2}}}$$

E : satellite elevation angle;
r : earth radius;
h : height of the ionosphere shell, normally 450 km.



(Source: Skone 2009)

Propagation-related correction – ionosphere

✓ **Recall that the first-order ionosphere effects are inversely proportional to the frequency**

$$\begin{cases} d_{iono}{}^{z}{}_{phase} = -\dfrac{40.3}{f^2} VTEC \\ d_{iono}{}^{z}{}_{code} = \dfrac{40.3}{f^2} VTEC \end{cases}$$

✓ **Linear combination between dual frequencies can remove the first-order ionosphere effect**

$$\begin{cases} \dfrac{f_1^2}{f_1^2 - f_2^2} d^z_{iono\ phase\_L1} - \dfrac{f_2^2}{f_1^2 - f_2^2} d^z_{iono\ phase\_L2} = \dfrac{f_1^2}{f_1^2 - f_2^2}\left(-\dfrac{40.3}{f_1^2} VTEC\right) - \dfrac{f_2^2}{f_1^2 - f_2^2}\left(-\dfrac{40.3}{f_2^2} VTEC\right) = 0 \\ \dfrac{f_1^2}{f_1^2 - f_2^2} d^z_{iono\,code\_L1} - \dfrac{f_2^2}{f_1^2 - f_2^2} d^z_{iono\,code\_L2} = \dfrac{f_1^2}{f_1^2 - f_2^2}\left(\dfrac{40.3}{f_1^2} VTEC\right) - \dfrac{f_2^2}{f_1^2 - f_2^2}\left(\dfrac{40.3}{f_2^2} VTEC\right) = 0 \end{cases}$$

Ionosphere-free code and phase linear combination

(Source: Skone 2009)

Propagation-related correction - Summary

Summary of troposphere/ionosphere effects, and corresponding calibration methods

	Troposphere	Ionosphere
Region	0~40 km	45~1500 km
Mechanism	Neutral particle; no charge	Ionized air; charge
Magnitude	A few meters	Up to tens of meters
Dispersive property	Frequency independent	Frequency dependent
Calibration method	• ZHD: modeling; • ZWD: estimation.	• External VTEC products; • Linear combination.

Receiver-related correction

Receiver-related errors can be further classified into two groups:

✓ **Observation related**
- Receiver antenna phase center offset
- Sagnac effect
- Receiver clock error

✓ **Site displacements**
- Solid earth tides
- Polar tides
- Ocean loading
- Earth rotation parameters

Receiver-related correction – Receiver antenna phase center offset

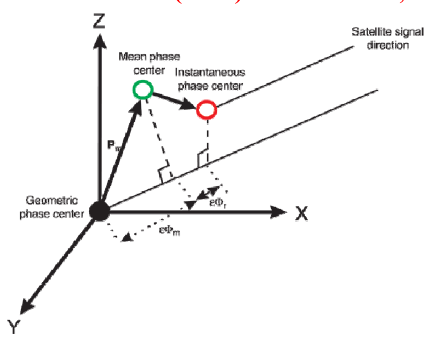
✓ **Consistent with satellite antenna phase center correction models**

✓ **Two receiver absolute antenna calibration methods**
- **Anechoic chamber & Antenna calibration robot.**

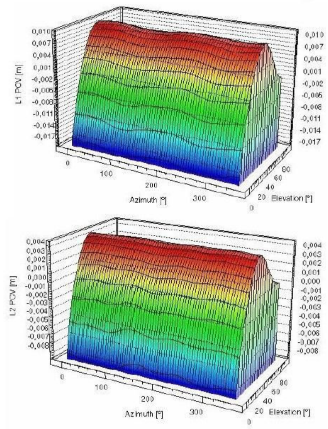
✓ **Latest receiver antenna phase center corrections are available at ftp://igs.org/igscb/station/general/igs08.atx.**

Receiver-related correction – Receiver antenna phase center offset

✓ **Receiver absolute antenna phase center corrections**
- **Phase center offset (PCO);**
- **Elevation- and azimuth-dependent Phase center variation (PCV) on L1 and L2;**



Source: Schmid 2010



GPS L1 and L2 PCV TPSCR.G3 TPSH

Source: Schmitz et al. 2008

Receiver-related correction – Receiver antenna phase center offset

Example

- Receiver antenna "ASH701941.B SCIS"
- Elevation: 32 degree; azimuth: 172 degree

Procedure

First, search antenna PCOs in the ENU directions for L1 and L2;
Second, the tabulated receiver PCVs of four grid points around the ele/azi angles are obtained;
Third, a bi-linear interpolation is applied to derive the ele- and azi-dependent receiver PCVs.

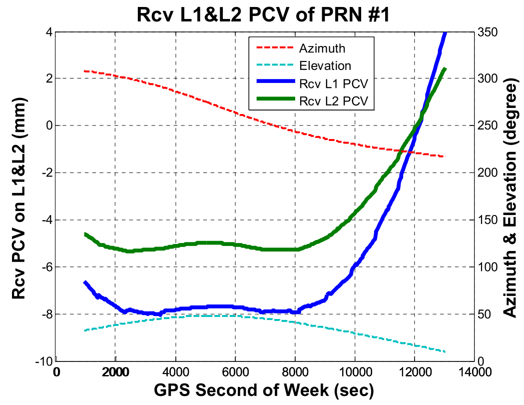
Result

Receiver antenna PCO and PCV corrections for "ASH701941.B SCIS"

Frequency	PCO in ENU (mm)	PCV (mm)		
		Tabulated values		Interpolated value
L1	*-0.60/0.24/85.11*	-9.54	-7.74	*-8.50*
		-9.64	-7.83	
L2	*-0.74/0.87/119.72*	-5.44	-4.61	*-4.49*
		-5.43	-4.62	

Receiver-related correction – Receiver antenna phase centre offset

- ✓ **Rcv PCVs on L1 and L2 are different;**
- ✓ **Consistent rcv PCO are used to calibrate rcv coordinates similar to ant height.**



Rcv L1&L2 PCV corrections of PRN #1

Receiver-related correction – Sagnac effect

✓ **Earth rotation during the signal transmitting period prevents the clock synchronization for the satellite and the receiver. This time issue is called sagnac effect which can amount up to hundreds of nanoseconds. Recall that one nanosecond time error leads to 30 cm distance error, the sagnac effect must be calibrated for cm-level PPP applications.**

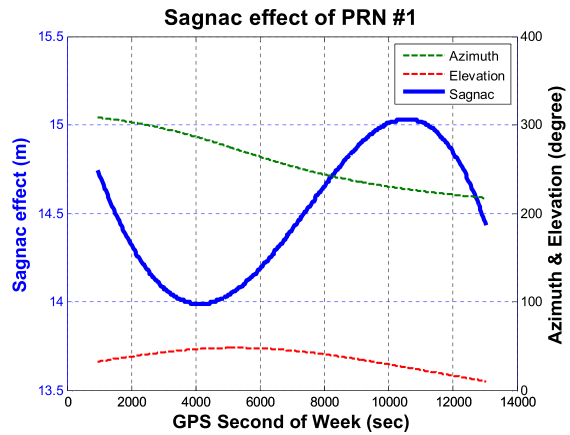
✓ **The GPS sagnac effect can be corrected by**

$$\Delta t_{sagnac} = \frac{v \cdot R}{c^2}$$

where

c is the speed of line in vacuum;
v is the receiver velocity in the ECI frame;
R is the vector between the satellite and the receiver.

Receiver-related correction – Sagnac effect



Sagnac effect correction of PRN #1

Receiver-related correction – Receiver clock

- ✓ **Receiver clock error**
  - Low-cost clock
  - Have a big drift, e.g. 1000 ns every second
  - Significant Clock offset
- ✓ **Measurement noise**
  - Random error
  - For high precision GPS receivers:
    - Code measurement noise level is at decimeter
    - Phase measurement noise level is at millimeter
- **For GPS positioning**
  - Receiver clock error is estimated;
  - Measurement noise is assumed to be random noise.

Receiver-related correction – Multipath

- ✓ **Multipath**
  - Caused by reflected signals from surfaces near the satellite/receiver
  - Reflected signals interfere with or be mistaken for the direct signal from the satellite
- ✓ **Multipath effect at GPS satellite**
  - Satellite antenna reflections
  - Usually not considered
- ✓ **Multipath effects at GPS receiver**
  - Reflective environments around the receiver
  - Up to tens of meters in code measurements
  - cm level in phase measurements
  - Greater influence on measurements to low elevation satellites

Receiver-related correction – Solid earth tide

✓ Sold earth tide has a permanent and periodical tidal displacement on the reference point attached to the earth crust.

✓ Generally speaking, the periodical vertical and horizontal site displacement can be represented by the spherical harmonics of degree and order 9 (IERS Conventions 2010).

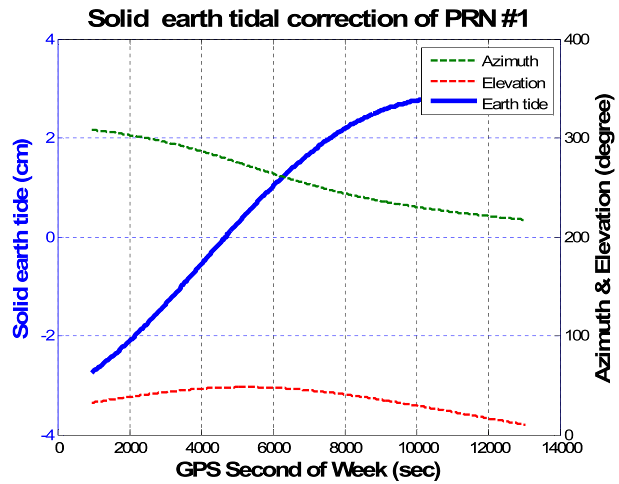
✓ The tidal effects can reach about 30 cm in the radial and 5 cm in the horizontal directions (Kouba 2009).

✓ The IERS-recommended solid earth tide correction is

$$\Delta\vec{r} = \sum_{j=2}^{3}\frac{GM_j}{GM}\frac{r^4}{R_j^3}\{[3l_2(\hat{R}_j\cdot\hat{r})]\hat{R}_j + [3(\frac{h_2}{2}-l_2)(\hat{R}_j\cdot\hat{r})^2-\frac{h_2}{2}]\hat{r}\}$$
$$+[-0.025m\cdot\sin(\varphi)\cdot\cos(\varphi)\cdot\sin(\theta_g+\lambda)]\cdot\hat{r}$$

Quantity representations and source code available @
http://tai.bipm.org/iers/conv2010/conv2010_c8.html

Receiver-related correction – Solid earth tide



Solid earth tidal correction of PRN #1

Receiver-related correction – Polar tide

✓ The polar tides are caused by the changes of the earth's spin axis with respect to the earth body.

✓ This earth spin axis variation, i.e. polar motion, can lead to periodical variation to the site displacement.

✓ Unlike the high-frequency solid earth tides, the polar tides experience a much slower period, i.e. seasonal and Chandler (~430 days) periods. The effects of polar motion can reach up to 0.8 arc sec and, subsequently, 25 mm and 7 mm in the height and horizontal site coordinates (Kouba 2009).

✓ The IERS-recommended polar tide correction is

$$\begin{cases} \Delta\phi = -9\cos(2\phi)[(X_P-\bar{X}_P)\cos(\lambda)-(Y_P-\bar{Y}_P)\sin(\lambda)] \\ \Delta\lambda = 9\sin(\phi)[(X_P-\bar{X}_P)\sin(\lambda)+(Y_P-\bar{Y}_P)\cos(\lambda)] \\ \Delta h = -33\sin(2\phi)[(X_P-\bar{X}_P)\cos(\lambda)-(Y_P-\bar{Y}_P)\sin(\lambda)] \end{cases}$$

Quantity representations and source code available @
http://tai.bipm.org/iers/conv2010/conv2010_c7.html

Receiver-related correction – Ocean loading

✓ The ocean loading is caused by the load of the ocean tides on the underlying crust.

✓ The magnitude of the ocean loading is one order smaller than that of the solid earth tides.

✓ There is no permanent part and the period of the ocean loading is much slower, i.e. diurnal and semi diurnal.

✓ The IERS-recommended ocean loading is

$$\Delta c = \sum_j f_j A_{cj}\cos(\omega_j t + \chi_j + u_j - \Phi_{cj})$$

Quantity representations and source code available @
http://tai.bipm.org/iers/conv2010/conv2010_c7.html

Receiver-related correction – Earth orientation parameter

✓ The earth rotation parameters (ERPs) are generally used to correct the pole position for the transformation between the inertial reference frame and the terrestrial reference frame (ITRF) used by the precise orbit products.

✓ Similar to ocean loading effect, the sub-daily earth rotation parameter effect can be averaged out to nearly zero over a 24 hours period for position solutions (but not for zenith tropospheric delay and receiver clock estimates).

✓ The IERS-recommended ERP correction is

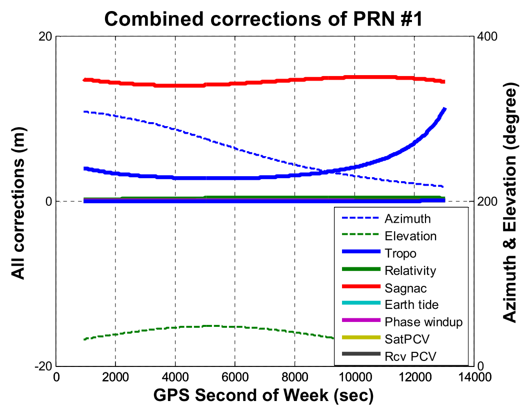
$$\delta X_P = \sum_{j=1}^{8} F_j^{Xp}\sin(\xi_j) + G_j^{Xp}\cos(\xi_j) \qquad \delta Yp = \sum_{j=1}^{8} F_j^{Yp}\sin(\xi_j) + G_j^{Yp}\cos(\xi_j)$$

$$\delta UT1 = \sum_{j=1}^{8} F_j^{UT1}\sin(\xi_j) + G_j^{UT1}\cos(\xi_j)$$

Quantity representations and source code available @
http://tai.bipm.org/iers/conv2010/conv2010_c8.html

Combined correction - Summary

✓ Satellite-related error: Relativistic effect + Phase wind-up + Sat PCV;
✓ Propagation-related error: troposphere delay;
✓ Receiver-related error: Sagnac effect + Earth tide + Rcv PCV;



Quantities	Magnitudes
Sagnac	14 ~ 15 m
Tropo	2 ~ 11 m
Relativity	0.15 ~ 0.45 m
Earth tide	-3 ~ 3 cm
Phase wind-up	2 ~ 4 cm
Sat PCV	-5 ~ 5 mm
Rcv PCV	-8 ~ 4 mm

Section 3

Precise GPS Product

- What is Precise GPS product?
- IGS GPS orbit and clock products
- IGS precise atmospheric products
- Precise orbit and clock towards real-time

What is Precise GPS Product?

- "Precise" is used in comparison to data broadcast from GPS satellites
- Precise GPS product include precise orbit, clock and atmospheric (troposphere and ionosphere) data
- Precise GPS product are usually determined using observations from a network of permanent reference GPS stations in regional or global scale, in both post-mission and real-time modes
- Precise GPS product are developed to support GPS applications with improved accuracy and integrity
- Precise GPS product are currently available from a number of organizations including International GNSS (IGS) Services, Jet Propulsion Laboratory (JPL), Natural Resources Canada (NRCan) etc.
- Precise GPS product are the foundation for the implementation of PPP methodology

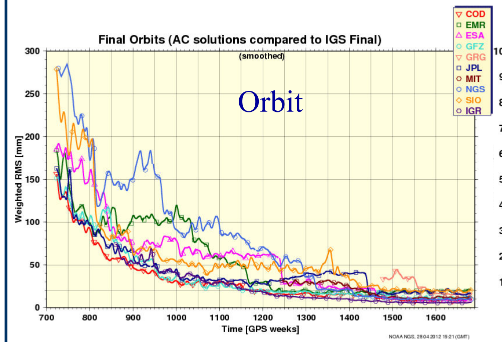
Precise Orbit and Clock Products from IGS

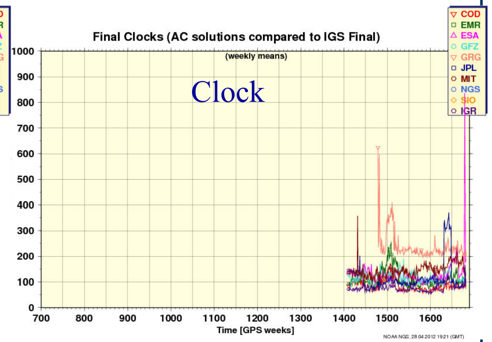
		Accuracy	Latency	Updates	Sample Interval
Broadcast*	Orbit	~ 100 cm	real-time	--	daily
	Clock	~ 3 ns RMS ~ 2.5 ns STD			
Ultra-Rapid (predicted half)	Orbit	~ 5 cm	real-time	at 03, 09, 15, 21 UTC	15 min
	Clock	~ 3 ns RMS ~ 1.5 ns STD			
Ultra-Rapid (observed half)	Orbit	~ 3 cm	3 ~ 9 hours	at 03, 09, 15, 21 UTC	15 min
	Clock	~ 150 ps RMS ~ 50 ps STD			
Rapid	Orbit	~ 2.5 cm	17 ~ 41 hours	at 17 UTC daily	15 min
	Clock	~ 75 ps RMS ~ 25 ps STD			5 min
Final	Orbit	~ 2.5 cm	12 ~ 18 days	Every Thursday	15 min
	Clock	~ 75 ps RMS ~ 20 ps STD			30s

(Available at http://igscb.jpl.nasa.gov/components/prods.html on 2012-04-30)
*: Broadcast ephemeris included for comparison purpose.

IGS Final Orbit and Clock Product

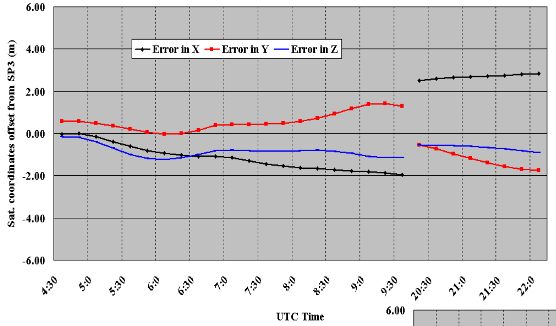
The IGS Final Products are the definitive set of GPS results provided for the general user community. They are designed to be fully self-consistent, within the noise level, and also consistent with International Terrestrial Reference Frame (ITRF) and the Conventions of the International Earth Rotation Service (IERS).

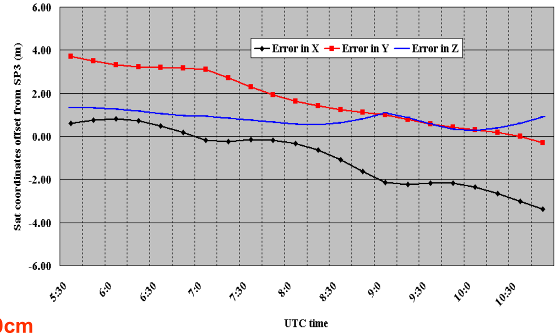




Source: http://acc.igs.org/ on 2012-04-30

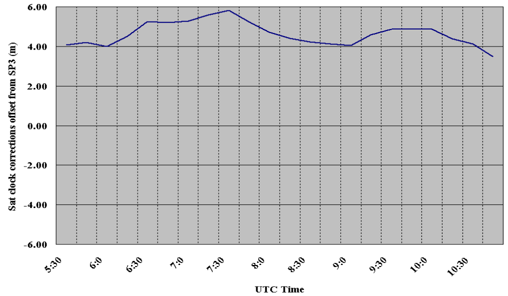
Coordinate Difference Precise vs Broadcast

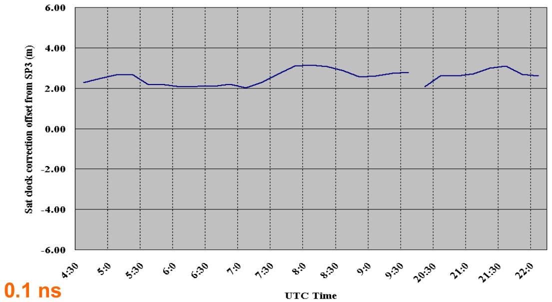




- Precise Orbit Accuracy < 5cm
- Broadcast Orbit Accuracy ~ 260cm

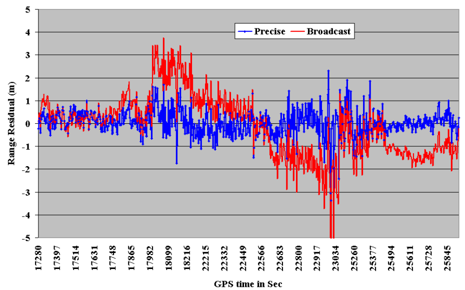
Clock Correction Difference Precise vs Broadcast





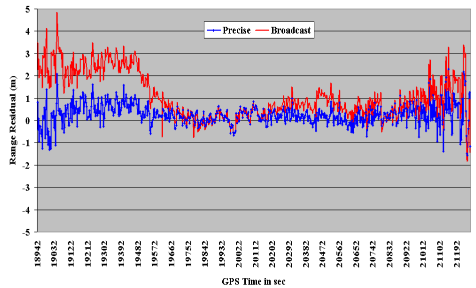
- Satellite Clock Corrections < 0.1 ns
- Broadcast Satellite Clock Corrections ~7 ns

Range Residual Analysis



Broadcast Versus Precise

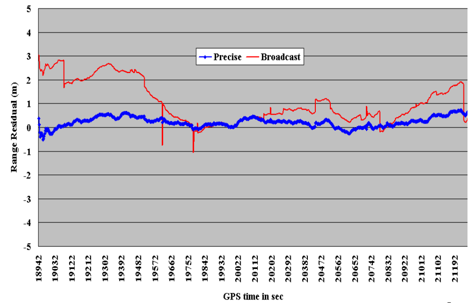
(code data, epoch by epoch)



Level of meters:

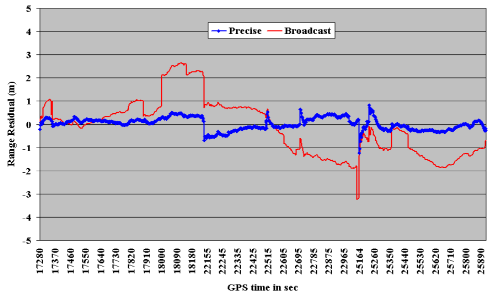
- Reflects P-Noise
- Orbit and Clock Errors

Range Residual Analysis



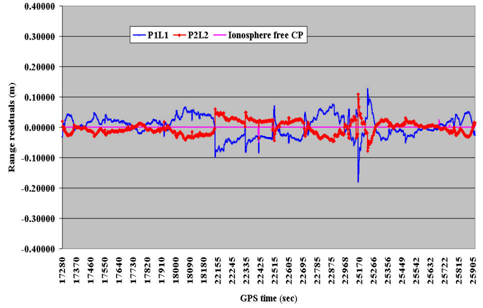
Broadcast Versus Precise

(Smoothed code, epoch by epoch)



Level of meters for broadcast
Decimeter level for precise

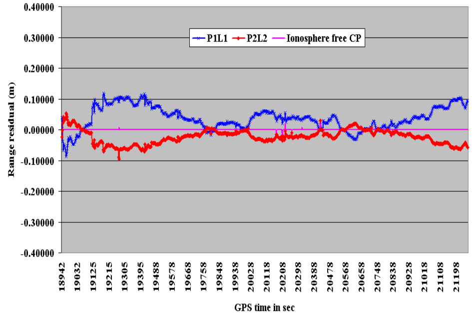
Phase Range Residual Analysis



P1+L1 and P2+L2

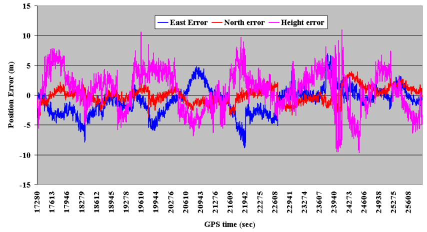
Centimeter level for P1+L1 and P2+L2

Millimeter level for ionosphere-free carrier phase



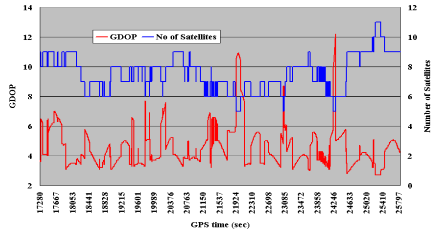
Position Accuracy Analysis

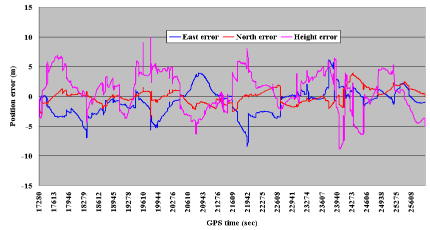
- Code Point Positioning Using Broadcast Ephemeris



Non-smoothed epoch by epoch

	East (m)	North (m)	Height (m)	Smoothing
Mean	-0.94	0.01	0.83	No
RMS	2.49	1.28	3.48	No
Mean	-0.90	0.01	0.90	Yes
RMS	2.37	1.23	3.42	Yes



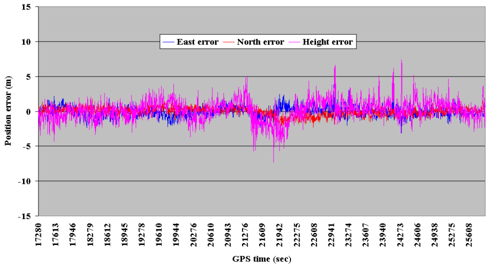


Smoothed epoch by epoch

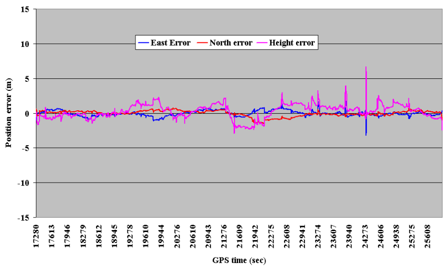
Position Accuracy Analysis

- Code Point Positioning Using Precise Orbit and Clock

Accuracy increased
RMS below 1m horizontal
RMS 1-1.5 for vertical



Non-smoothed epoch by epoch

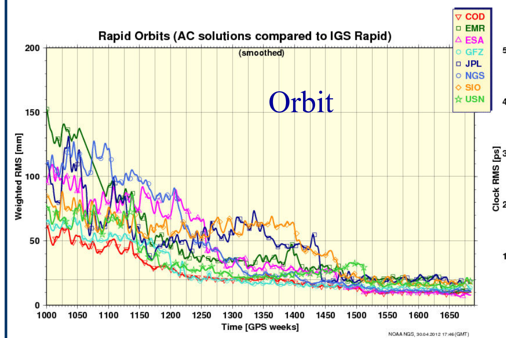


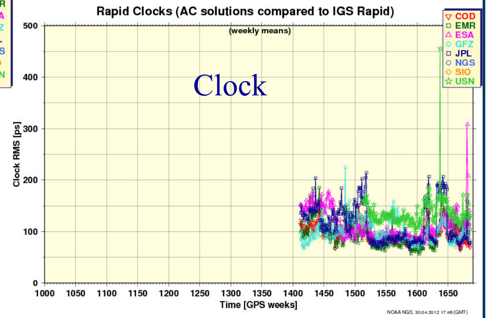
Smoothed epoch by epoch

	East (m)	North (m)	Height (m)	Smoothing
Mean	-0.06	-0.06	0.18	No
RMS	0.72	0.58	1.50	No
Mean	-0.04	-0.04	0.28	Yes
RMS	0.40	0.42	1.06	Yes

IGS Rapid Orbit and Clock Product

The IGS Rapid Products are intended as a near-definitive set of products for users unable to wait for the Final Products. Currently, the accuracy is almost the same as the Finals. The latency is reduced to ~ 17 hours.

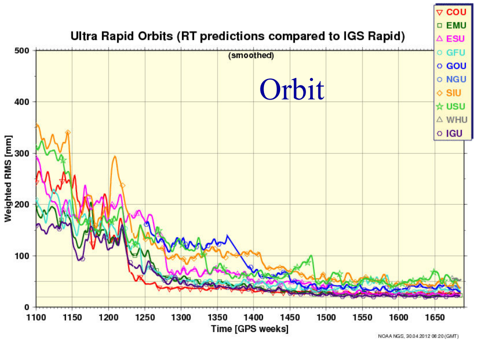


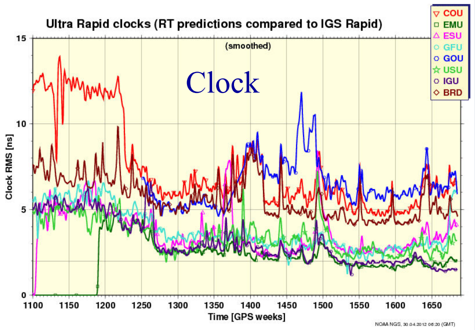


Source: http://acc.igs.org/ on 2012-04-30

IGS Ultra-Rapid Orbit and Clock Product

The IGS Ultra-Rapid Products are intended as a set of GPS products for high-accuracy real-time users. They are forward extrapolations using the latest and best observational results available. The goal is an orbit accuracy better than 30 cm, preferably better than 20 cm.

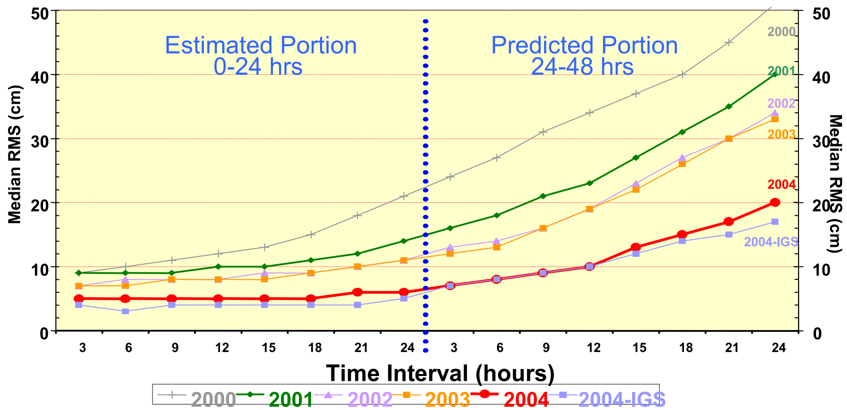




Source: http://acc.igs.org/ on 2012-04-30

IGS Ultra-Rapid Orbit and Clock Product

IGS Ultra Rapid Orbit Median RMS when compared to IGR



Precise Atmospheric Products from IGS (1/3)

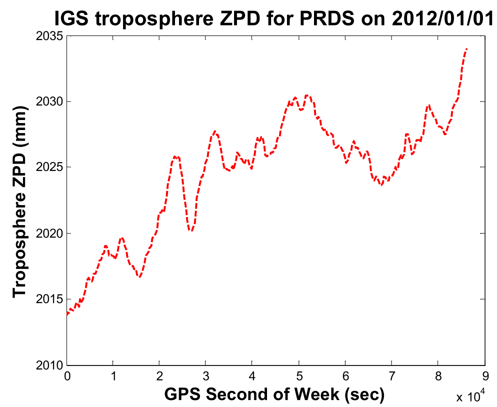
Product type	Accuracy	Latency	Updates	Sample Interval
Final tropospheric zenith path delay	4 mm	< 4 weeks	weekly	2 hours
Ultra-Rapid tropospheric zenith path delay	6 mm	2-3 hours	every 3 hours	1 hour
Final ionospheric TEC grid	2-8 TECU	~11 days	weekly	2 hours
Rapid ionospheric TEC grid	2-9 TECU	<24 hours	daily	2 hours

(Available at http://igscb.jpl.nasa.gov/components/prods.html on 2012-04-30)

Precise Atmospheric Products from IGS (2/3)

IGS troposphere ZPD product

- Software: GIPSY
- Data time span: 24 h
- Data rate: 5 min
- Elevation angle cutoff: 7◦
- Mapping function: Niell (1996)
- A priori: Hydrostatic delay based on altitude (2.3 m at sea level), and 0.1 m for the wet delay

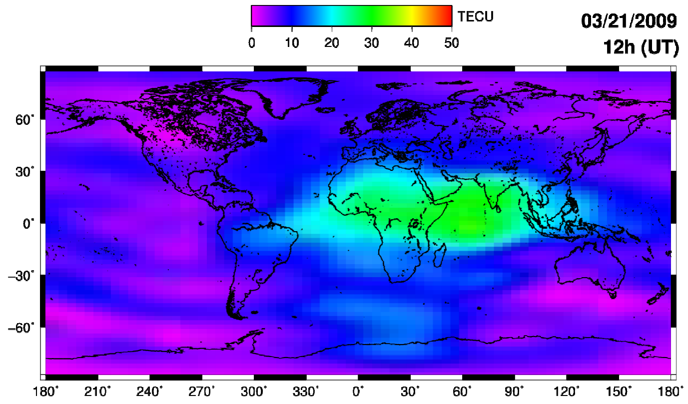


IGS troposphere ZPD for station PRDS on 2012/01/01

Precise Atmospheric Products from IGS (3/3)

IGS ionosphere VTEC product

- Global 2.5*2.0 degree grid
- Data rate: 2 hour



Global Ionospheric Map on 12h of 2009/03/21

Source: http://gnss.be/ionosphere_tutorial.php

Precise Orbit and Clock Products Towards Real-Time (1/10)

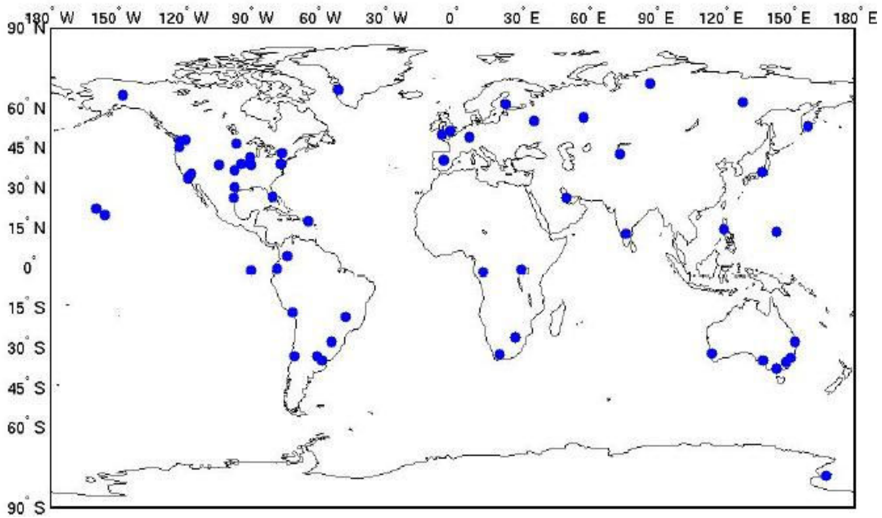
Real-time satellite orb/clk correction messages before 2007

- JPL real-time corrections (IGDG)
- NRCan phase solution corrections (GPS•C)

IGS Real-time Pilot Project since 2007

- Project released in June 2007
- IGS real-time permanent tracking station data streams as infrastructure
- RTCM 3.0 types 1001-1020 were defined to support real-time data streams
- IGS analysis centers (AC) estimate real-time sat orb/clk correction
- RTCM-SSR types 1057-1068 were defined to support real-time sat correction message streams

Precise Orbit and Clock Products Towards Real-Time (2/10)



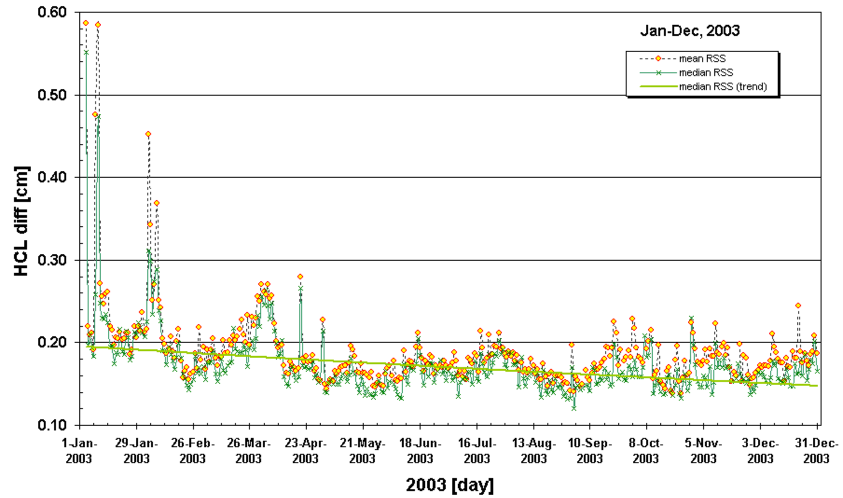
- Stations: ~70
- Accuracy:
  Orbits:~20 cm
  Clocks:~0.5 ns
- Latency:
  ~4 seconds
- Update interval:
  1 second

JPL real-time corrections (IGDG)

Source: Ronald Muellerschoen (2004)

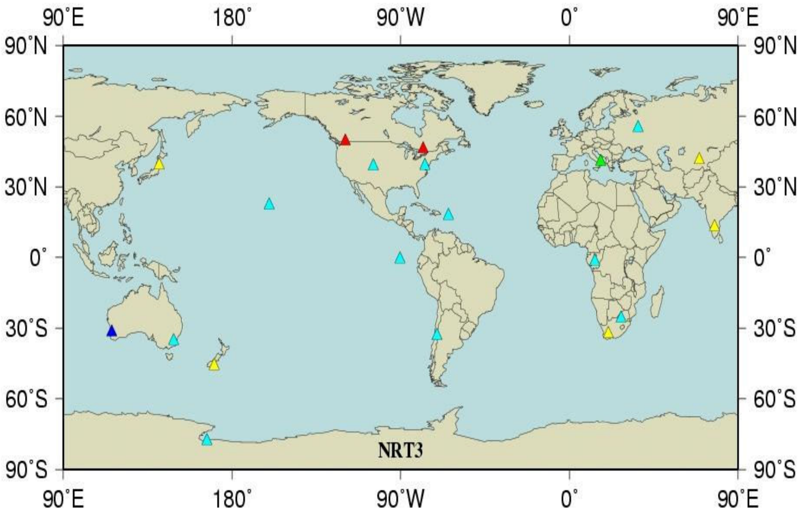
Precise Orbit and Clock Products Towards Real-Time (3/10)

JPL RTG vs FLINN orbit diff:
daily mean, median RSS of HCL differences for all SVN



JPL real-time corrections (IGDG)

Precise Orbit and Clock Products Towards Real-Time (4/10)



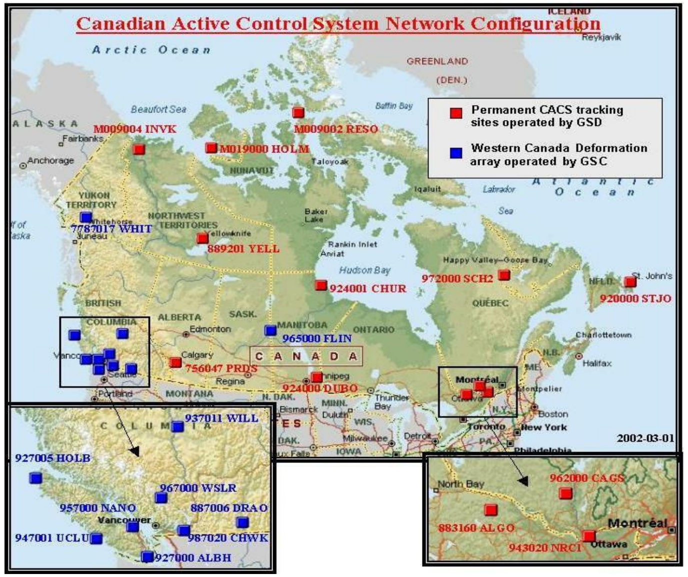
- Stations: ~20
- Accuracy:
  Orbits:~10 cm
  Clocks:~1 ns
- Latency:
  ~several hours
- Update interval:
  2 second

NRCan phase solution corrections (GPS•C)

Source: Paul Collins (2004)

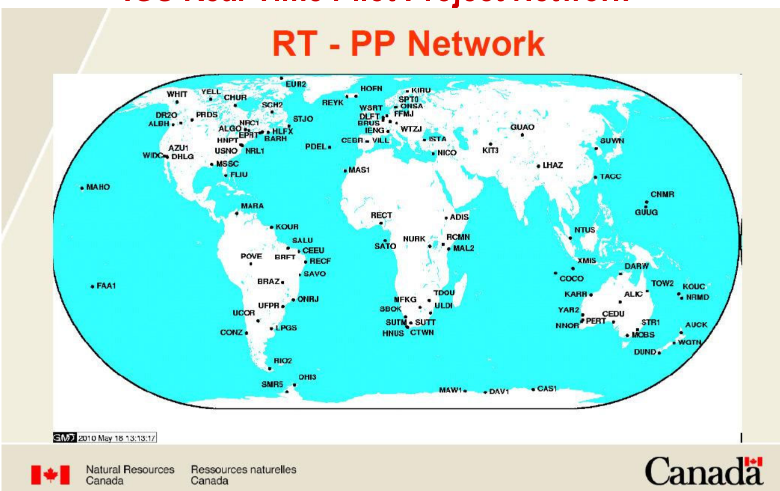
Precise Orbit and Clock Products Towards Real-Time (5/10)

Canada Real-Time Network



Precise Orbit and Clock Products Towards Real-Time (6/10)

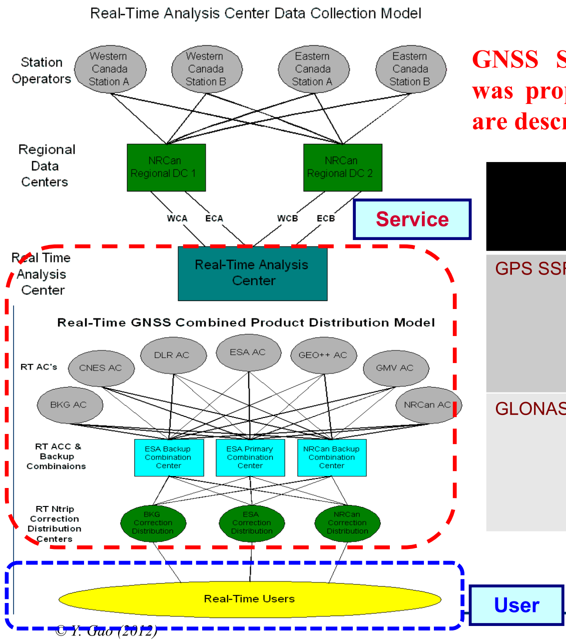
IGS Real-Time Pilot Project Network



64 Contributors **36 Countries** **185 Streams**

Source: Ken MacLeod and Mark Caissy (2010)

Precise Orbit and Clock Products Towards Real-Time (7/10)



GNSS Space State Representative (SSR) was proposed in 2005. The SSR messages are described by RTCM type 1057-1068.

	RTCM type	Description
GPS SSR	1057 1058 1059 1060 1061 1062	Orbit correction Clock correction Code biases Orbit/clk correction Sat URA High-rate clock correction
GLONASS SSR	1063 1064 1065 1066 1067 1068	Orbit correction Clock correction Code biases Orbit/clk correction Sat URA High-rate clock correction

Precise Orbit and Clock Products Towards Real-Time (8/10)

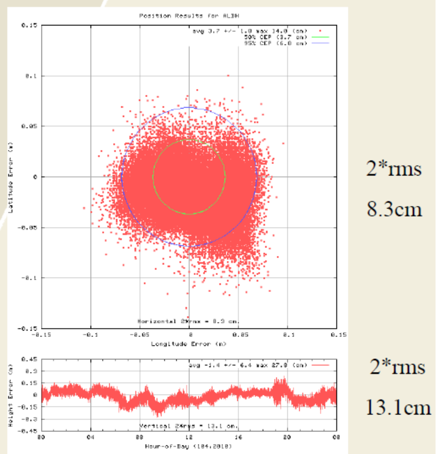
IGS real-time sat orbit/clock correction accuracy

Average of last 7 days (June 17-23/10)

Analysis Centre	Orbit RMS (mm)	Sat. Clock RMS (ns)	Sat Clock Sigma (ns)
Combination		0.17	0.11
RT - Combination	40.95	0.17	0.11
BKG	46.5	0.22	0.11
DLR	49.9	0.22	0.14
ESOC	47.9	0.21	0.11
NRCan	37.9	0.22	0.11
GMV	78.7	0.43	0.16
GFZ	53.0	0.60	0.35
TUW	264.2	0.77	0.54

18

Natural Resources Canada Ressources naturelles Canada Canada



Positioning accuracy

Source: Ken MacLeod and Mark Caissy(2010)

Precise Orbit and Clock Products Towards Real-Time (9/10)

Status of real-time orbit/clock correction

Several agencies can routinely provide real-time GNSS satellite orbit/clock correction messages. Currently available streams can be found at http://igs.bkg.bund.de/ntrip/orbits?PHPSESSID=26ac109beca8edcd512e3e24146f463e.

Definition of real-time orbit/clock correction messages has been incorporated into RTCM 10403.1 with Amendment 5 @ https://ssl29.pair.com/dmarkle/puborder.php?show=3.

It is feasible for users to perform real-time PPP!

Precise Orbit and Clock Products Towards Real-Time (9/10)

Working Together

International GNSS Service
Formerly the International GPS Service

http://igscb.jpl.nasa.gov/index.html

Real Time Working Group
Home Activities FAQ's Request Info Pilot Project

http://www.rtigs.net/rtigswg/index.php

IAG SC 4.5: High-Precision GNSS Algorithm and Applications

International Association of Geodesy
COMMISSION 4: Positioning & Applications

http://people.ucalgary.ca/~point/iag.html

Section 4

Data Processing in PPP

- Measurements
- Observation combinations and models
- Estimation method and stochastic modeling
- Carrier phase ambiguity in PPP
- Float ambiguity convergence analysis
- Single-frequency PPP

Measurements (Un-differenced)

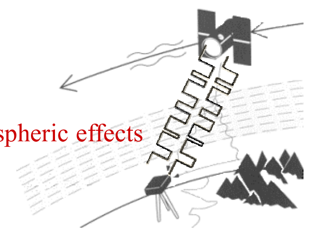
Code measurement (pseudorange):

$$P(Li) = \rho + c(dt - dT) + d_{orb} + d_{trop} + d_{ion/Li} + b^{r}_{P(L_i)} - b^{s}_{P(L_i)} + d_{mult/P(Li)} + \varepsilon(P(Li))$$

where

$i = 1, 2$

$P(Li)$	is the measured pseudorange on Li (m);
ρ	is the true geometric range (m);
c	is the speed of light (m/s);
dt	is the satellite clock error (s);
dT	is the receiver clock error (s);
d_{orb}	is the satellite orbit error (m);
d_{trop}	is the tropospheric delay (m);
$d_{ion/Li}$	is the ionospheric delay on Li (m);
$b^{r}_{P(Li)}$	is the receiver bias (m);
$b^{s}_{P(Li)}$	is the satellite bias;
$d_{mult/P(Li)}$	is the multipath effect in the measured pseudorange on Li (m);
$\varepsilon(.)$	is the measurement noise (m).



Measurements (Un-differenced)

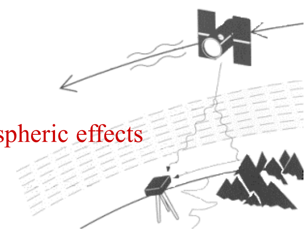
Carrier Phase Measurement:

$$\Phi(Li) = \rho + c(dt - dT) + d_{orb} + d_{trop} - d_{ion/Li} + \lambda_i N_i + \lambda_i(\phi_r(t_0, Li) - \phi_s(t_0, Li)) + d_{mult/\Phi(Li)} + \varepsilon(\Phi(Li))$$

where

$i = 1, 2$

$\Phi(Li)$	is the measured carrier phase on Li (m);
ρ	is the true geometric range (m);
c	is the speed of light (m/s);
dt	is the satellite clock error (s);
dT	is the receiver clock error (s);
d_{orb}	is the satellite orbit error (m);
d_{trop}	is the tropospheric delay (m);
$d_{ion/Li}$	is the ionospheric delay on Li (m);
λ_i	is the wavelength on Li (m);
N_i	is the integer phase ambiguity on Li (cycle);
$\phi_r(t_0, Li)$	is the initial phase of the receiver oscillator;
$\phi_s(t_0, Li)$	is the initial phase of the satellite oscillator;
$d_{mult/\Phi(Li)}$	is the multipath effect in the measured pseudorange on Li (m);
$\varepsilon(.)$	is the measurement noise (m).



Observation Combinations and Models for PPP

Traditional model

- **Ionosphere-free code combination**

$$P_{IF} = \rho - cdT + d_{trop} + d_{mult/(P1+P2)} + \varepsilon$$

- **Ionosphere-free phase combination**

$$\Phi_{IF} = \rho - cdT + d_{trop} + \frac{cf_1 N_1 - cf_2 N_2}{f_1^2 - f_2^2} + d_{mult/\Phi(L_1+L_2)} + \varepsilon$$

Positioning Unknowns

- Position coordinates
- Combined L1 and L2 ambiguities (float)
- Receiver clock
- Tropo parameters

Features

- Only float solution is possible
- Increased noise level in the combined ionosphere-free code/phase observations
- Integer property of the carrier phase ambiguity can not be exploited

Observation Combinations and Models for PPP

UofC model

- **Ionosphere-free code and phase combination**

$$P_{IF,L1} = \rho - cdT + d_{trop} + 0.5\lambda_1 N_1 + 0.5d_{mult/P(L1)} + \varepsilon$$
$$P_{IF,L2} = \rho - cdT + d_{trop} + 0.5\lambda_2 N_2 + 0.5d_{mult/P(L2)} + \varepsilon$$

- **Ionosphere-free phase combination**

$$\Phi_{IF} = \rho - cdT + d_{trop} + \frac{cf_1}{f_1^2 - f_2^2} N_1 - \frac{cf_2}{f_1^2 - f_2^2} N_2 + d_{mult/\Phi(L_1+L_2)} + \varepsilon$$

Positioning unknowns

- Position coordinates
- Separate L1 and L2 ambiguities (float)
- Receiver clock
- Tropo parameters

Features

- Allow for simultaneous estimation of L1 and L2 ambiguities
- Reduced noise level for the new code observations

Observation Combinations and Models for PPP

Carrier Phase Smoothing

$$P_1^{SM}(n) = \frac{1}{m}P_1(n) + (1-\frac{1}{m})(P_1^{SM}(n-1) + \Phi_{IF}(n) - \Phi_{IF}(n-1))$$
$$= \rho - cdT + d_{trop} - 0.5\lambda_1 N_1 + \varepsilon(P_1^{SM}(n))$$

$$P_2^{SM}(n) = \frac{1}{m}P_2(n) + (1-\frac{1}{m})(P_2^{SM}(n-1) + \Phi_{IF}(n) - \Phi_{IF}(n-1))$$
$$= \rho - cdT + d_{trop} - 0.5\lambda_2 N_2 + \varepsilon(P_2^{SM}(n))$$

$$\Phi_{IF} = \rho - cdT + d_{trop} + \frac{cf_1}{f_1^2 - f_2^2}N_1 + \frac{cf_2}{f_1^2 - f_2^2}N_2 + \varepsilon(\Phi_{IF})$$

GPS Measurement Error Mitigation in PPP

Error Source	Mitigation Methods	Residual Error
Ionospheric error	Ionosphere-free combinations	Eliminated
Tropospheric error	Modeled	cm level
	Estimated	
Satellite orbit	Precise orbit product	cm level
Satellite clock	Precise clock product	sub-ns level
Receiver Clock	Estimated	

Model Comparison

	Traditional Model	UofC Model
Observations	Two per satellite, one code and one phase	Three per satellite: two code and one phase
Noise	Three times of the original code noise level	Approximately half of the original code noise level
Ambiguity	Combined L1/L2 ambiguity, float value	L1 and L2 ambiguity, pseudo-integer, leading to fixed solution
Unknown parameters	Three-dimension coordinates Trop wet zenith delay Rex clock offset N ambiguities	Three-dimension coordinates Trop wet zenith delay Rex clock offset 2×N ambiguities

Estimation Method: Kalman Filtering

Dynamic model: $\dot{x}(t) = Fx(t) + w(t)$ $\quad Q$

State vector (x), Dynamic matrix (F), Dynamic noise vector (w), Spectral density matrix (Q)

Discrete form: $x_k = \Phi_{k-1}x_{k-1} + w_{k-1}$

State vector, Transition matrix, Dynamic noise vector

Measurement model: $z_k = H_k x_k + v_k$

Measurement vector, Design matrix, Measurement noise

Stochastic model:

$$E(w_k w_j^T) = \begin{cases} Q_k & k = j \\ 0 & k \neq j \end{cases} \qquad E(v_k v_j^T) = \begin{cases} R_k & k = j \\ 0 & k \neq j \end{cases}$$

$$E(w_k v_j^T) = 0 \qquad Q_k = \int_0^{\Delta t} \Phi(t_k, \tau) Q(\tau) \Phi(t_k, \tau)^T d\tau$$

Estimation Method: Kalman Filtering

Prediction:

$$\hat{x}_k^- = \Phi_{k-1}\hat{x}_{k-1}^{(+)}$$

$$P_k^- = \Phi_{k-1}P_{k-1}^+\Phi_{k-1}^T + Q_{k-1}$$

Updating/filtering:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k[z_k - H_k\hat{x}_k^-]$$

$$P_k^+ = [I - K_kH_k]P_k^-$$

$$K_k = P_k^-H_k^T[H_kP_k^-H_k^T + R_k]^{-1}$$

- Recursive computation
- Suitable for static and kinematic positioning
- Real-time or post-mission data processing

Estimation Method: Stochastic Modeling

➢ Static Positioning

- Position (latitude, longitude, height): constant
- L1 and L2 ambiguities (float): constant
- Receiver clock (one for a receiver station): random walk or first order Gauss-Markov (GM)
- Zenith tropospheric delay (one for a station): random walk
- Zenith tropospheric delay gradients: random walk

➢ Kinematic Positioning

- Position (latitude, longitude, height): random walk or first order Gauss-Markov (GM)
- Velocity (north, east and up): random walk or first order Gauss-Markov
- L1 and L2 ambiguities (float): constant
- Receiver clock (one for a receiver station): random walk or first order Gauss-Markov (GM)
- Zenith tropospheric delay (one for a station): random walk
- Zenith tropospheric delay gradients: random walk

Stochastic Modeling: First Order GM Process

- Dynamic system model:

$$\dot{x}(t) = -\beta x(t) + w(t) \qquad Q$$

$$x_k = e^{-\beta\Delta t}x_{k-1} + w_{k-1} \qquad Q_k = \frac{1}{2\beta}\left(1 - e^{-2\beta\Delta t}\right)Q$$

where β is the inverse of the correlation length. A larger value of β, i.e., a short correlation length, allows a large change in the state vector from one epoch to the next; a small value of β, i.e., a longer correlation length, describes a strong correlation between subsequent epochs and will allow little variation in the state vector.

- Velocity and receiver clock drift modeled as a first order GM process
- Dynamic system model:

$$\begin{bmatrix} \delta\dot{\varphi} \\ \delta\dot{\lambda} \\ \delta\dot{h} \\ \delta\dot{v}_N \\ \delta\dot{v}_E \\ \delta\dot{v}_h \\ \delta d\dot{T} \\ \delta d\ddot{T} \end{bmatrix} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{R} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R\cos\varphi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\beta v_N & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta v_E & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta v_h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{d\dot{T}} \end{pmatrix} \begin{bmatrix} \delta\varphi \\ \delta\lambda \\ \delta h \\ \delta v_N \\ \delta v_E \\ \delta v_h \\ \delta dT \\ \delta d\dot{T} \end{bmatrix} + w(t) \qquad Q = \mathrm{diag}(0, 0, 0, q_{vN}, q_{vE}, q_{vh}, 0, q_{d\dot{T}})$$

Stochastic Modeling: First Order GM Process

- Transition matrix

$$\Phi_k = \begin{pmatrix} 1 & 0 & 0 & \frac{1-e^{-\beta v_N\Delta t}}{\beta_{v_N}R} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1-e^{-\beta v_E\Delta t}}{\beta_{v_E}R\cos\varphi} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1-e^{-\beta v_h\Delta t}}{\beta_{v_h}} & 0 & 0 \\ 0 & 0 & 0 & e^{-\beta v_N\Delta t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\beta_{v_E}\Delta t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\beta v_h\Delta t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1-e^{-\beta_{v_{d\dot{T}}}\Delta t}}{\beta_{v_{d\dot{T}}}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-\beta_{v_{d\dot{T}}}\Delta t} \end{pmatrix}$$

Stochastic Modeling: First Order GM Process

- **Dynamic system covariance matrix:** $Q_k = \left(C_{ij}\right)_{8x8}$

$$C_{11} = \frac{q_{v_N}}{\beta_{v_N}^2}\{\Delta t - \frac{2}{\beta_{v_N}}(1-e^{-\beta_{v_N}\Delta t}) + \frac{1}{2\beta_{v_N}}(1-e^{-2\beta_{v_N}\Delta t})\}$$

$$C_{22} = \frac{q_{v_E}}{\beta_{v_E}^2}\{\Delta t - \frac{2}{\beta_{v_E}}(1-e^{-\beta_{v_E}\Delta t}) + \frac{1}{2\beta_{v_E}}(1-e^{-2\beta_{v_E}\Delta t})\}$$

$$C_{33} = \frac{q_{v_h}}{\beta_{v_h}^2}\{\Delta t - \frac{2}{\beta_{v_h}}(1-e^{-\beta_{v_h}\Delta t}) + \frac{1}{2\beta_{v_h}}(1-e^{-2\beta_{v_h}\Delta t})\}$$

$$C_{44} = \frac{q_{v_N}}{2\beta_{v_E}}(1-e^{-2\beta_{v_E}\Delta t})$$

$$C_{55} = \frac{q_{v_E}}{2\beta_{v_E}}(1-e^{-2\beta_{v_E}\Delta t})$$

$$C_{66} = \frac{q_{v_h}}{2\beta_{v_h}}(1-e^{-2\beta_{v_h}\Delta t})$$

$$C_{77} = \frac{q_{d\dot{T}}}{\beta_{d\dot{T}}^2}\{\Delta t - \frac{2}{\beta_{d\dot{T}}}(1-e^{-\beta_{d\dot{T}}\Delta t}) + \frac{1}{2\beta_{d\dot{T}}}(1-e^{-2\beta_{d\dot{T}}\Delta t})\}$$

$$C_{88} = \frac{q_{d\dot{T}}}{2\beta_{d\dot{T}}}(1-e^{-2\beta_{d\dot{T}}\Delta t})$$

$$C_{14} = C_{41} = \frac{q_{v_N}}{\beta_{v_N}}\{\frac{1}{\beta_{v_N}}(1-e^{-\beta_{v_N}\Delta t}) - \frac{1}{2\beta_{v_N}}(1-e^{-2\beta_{v_N}\Delta t})\}$$

$$C_{25} = C_{52} = \frac{q_{v_E}}{\beta_{v_E}}\{\frac{1}{\beta_{v_E}}(1-e^{-\beta_{v_E}\Delta t}) - \frac{1}{2\beta_{v_E}}(1-e^{-2\beta_{v_E}\Delta t})\}$$

$$C_{36} = C_{63} = \frac{q_{v_h}}{\beta_{v_h}}\{\frac{1}{\beta_{v_h}}(1-e^{-\beta_{v_h}\Delta t}) - \frac{1}{2\beta_{v_h}}(1-e^{-2\beta_{v_h}\Delta t})\}$$

$$C_{78} = C_{87} = \frac{q_{d\dot{T}}}{\beta_{d\dot{T}}}\{\frac{1}{\beta_{d\dot{T}}}(1-e^{-\beta_{d\dot{T}}\Delta t}) - \frac{1}{2\beta_{d\dot{T}}}(1-e^{-2\beta_{d\dot{T}}\Delta t})\}$$

Stochastic Modeling: Random Walk Process

- **Dynamic system model:**

$$\dot{x}(t) = w(t) \qquad Q$$

$$x_k = x_{k-1} + w_{k-1} \qquad Q_k = Q\Delta t$$

- **Tropospheric delay as random a walk process**

$$x_k = x_{k-1} + w_{k-1} \qquad Q_k = q_{trop}\Delta t$$

- **Receiver clock drift modeled as a random walk process**

$$\begin{bmatrix} \delta d\dot{T} \\ \delta d\ddot{T} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta dT \\ \delta d\dot{T} \end{bmatrix} + w(t) \qquad Q = \begin{bmatrix} 0 & 0 \\ 0 & q_{d\dot{T}} \end{bmatrix}$$

$$\begin{bmatrix} \delta dT_k \\ \delta d\dot{T}_k \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \delta dT_{k-1} \\ \delta d\dot{T}_{k-1} \end{bmatrix} + w_k \qquad Q_k = \begin{bmatrix} \frac{1}{3}q_{dT}\Delta t^3 & \frac{1}{2}q_{dT}\Delta t^2 \\ \frac{1}{2}q_{dT}\Delta t^2 & q_{dT}\Delta t \end{bmatrix}$$

Carrier Phase Ambiguity in PPP

- **Nature of ambiguity in PPP**

$$\Phi(Li) = \rho + c(dt - dT) + d_{orb} + d_{trop} - d_{ion/Li} + \lambda_i\{N_i + [\phi_r(t_0, Li) - \phi_s(t_0, Li)]\}$$
$$+ d_{mult/\Phi(Li)} + \varepsilon\,(\Phi(Li))$$

Receiver → Satellite → Non-zero initial phase bias → not integer any more !!!

Non-zero-initial phase offset is neither

- an observation error
- nor an equipment related phenomena

PPP ambiguities are Real-valued (float) in nature

Float Ambiguity Convergence

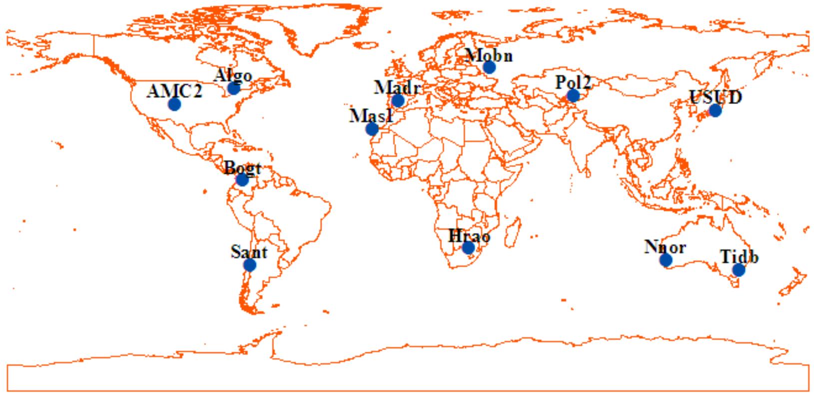
- **Position convergence depends on the convergence of the ambiguity parameters**
- **Float ambiguity estimates however need time to converge ranging from several tens of minutes to several hours**
- **Fast ambiguity convergence is required for real-time applications**
- **Integer ambiguities must be resolved to fully realize the accuracy of carrier phase observations: fixing the ambiguity to their integer values.**
  - Allow instantaneous position convergence
  - Instantaneous centimetre-level positioning accuracy

But ambiguity in PPP is not integer if the initial phase bias not eliminated.

Parameters that Affect Carrier Phase Ambiguity Convergence

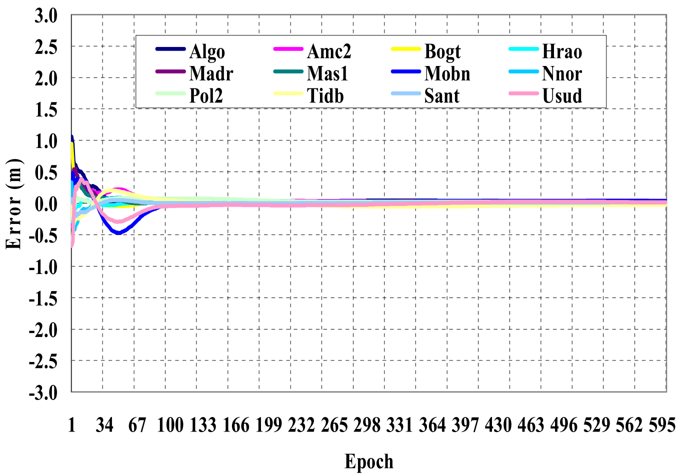
- Accuracy of observations
- Accuracy of precise orbit and clock products
- Accuracy of other correction products
- Accuracy of error mitigation
- Cycle slip detection
- Blunder detection and quality control
- Number of visible satellites
- Geometric strength of satellite configuration
- Measurement system dynamics
- Observation models
- Accuracy of stochastic modeling

Ambiguity Convergence Analysis



Test Stations

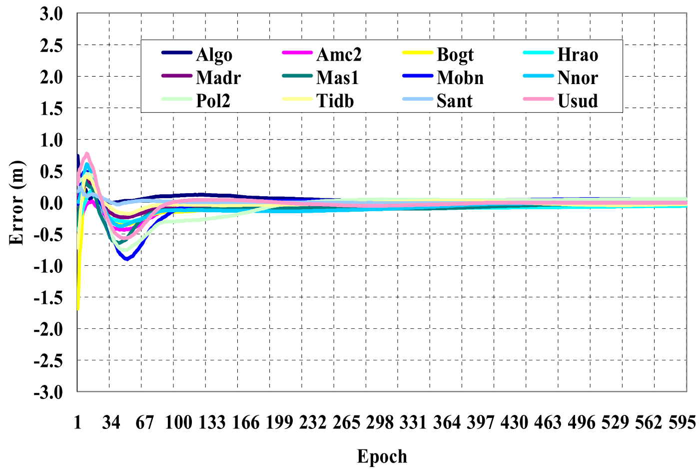
Ambiguity Convergence Analysis



Latitude Error

(Precise clock rate: 30 sec Observation data rate: 30 sec)

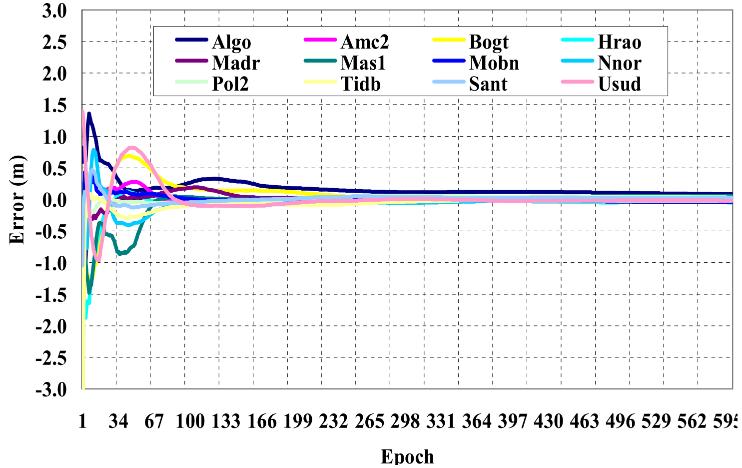
Ambiguity Convergence Analysis



Longitude Error

(Precise clock rate: 30 sec Observation data rate: 30 sec)

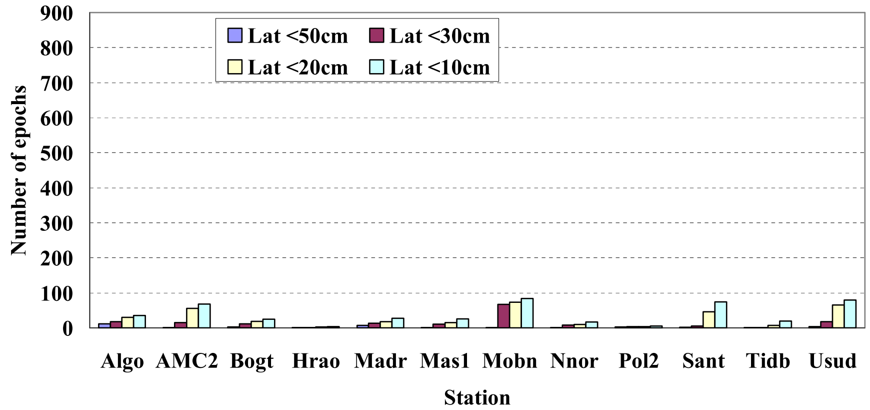
Ambiguity Convergence Analysis



Height Error

(Precise clock rate: 30 sec Observation data rate: 30 sec)

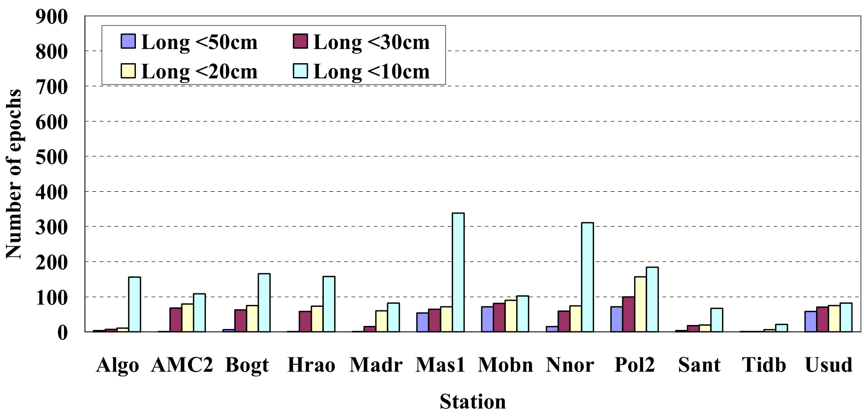
Ambiguity Convergence Analysis



Latitude Error

(Precise clock rate: 30 sec Observation data rate: 30 sec)

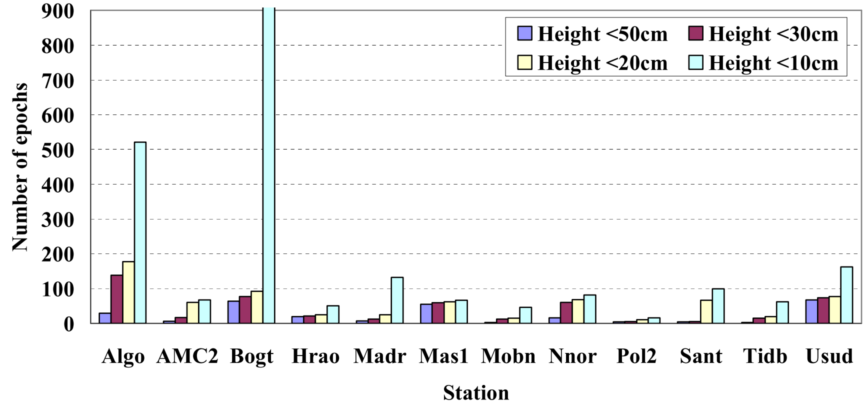
Ambiguity Convergence Analysis



Longitude Error

(Precise clock rate: 30 sec Observation data rate: 30 sec)

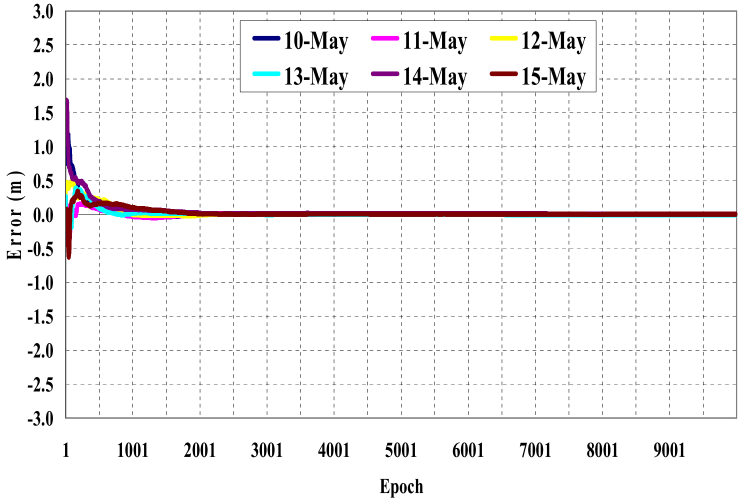
Ambiguity Convergence Analysis



Height Error

(Precise clock rate: 30 sec Observation data rate: 30 sec)

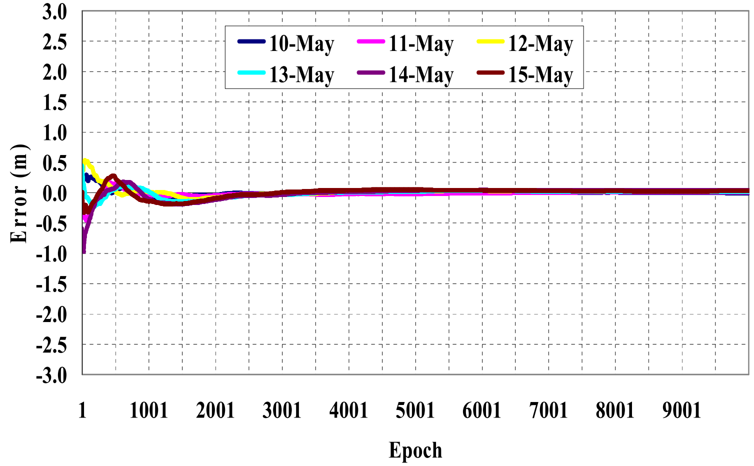
Ambiguity Convergence Analysis



Latitude Error

(Site: Calgary Precise clock rate: 30 sec Observation data rate: 1 sec)

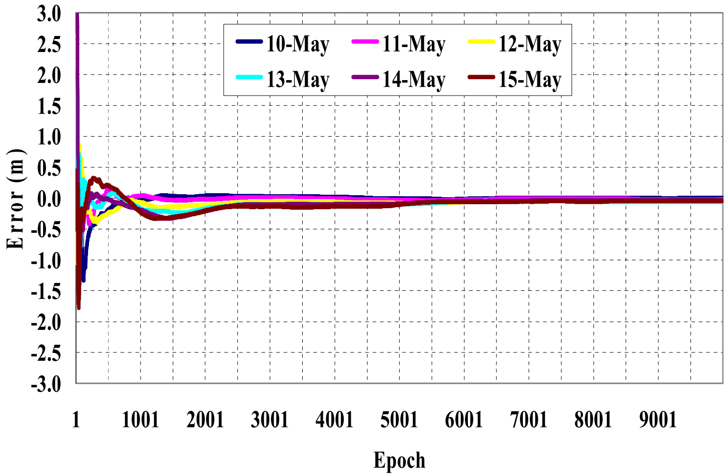
Ambiguity Convergence Analysis



Latitude Error

(Site: Calgary Precise clock rate: 30 sec Observation data rate: 1 sec)

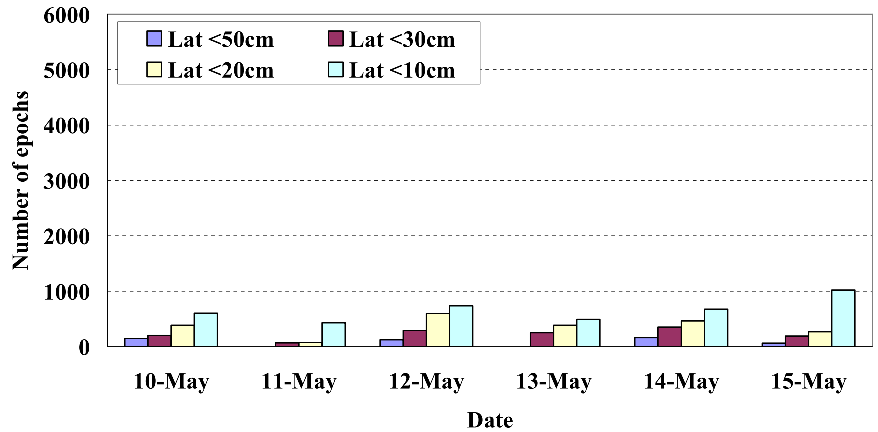
Ambiguity Convergence Analysis



Height Error

(Site: Calgary Precise clock rate: 30 sec Observation data rate: 1 sec)

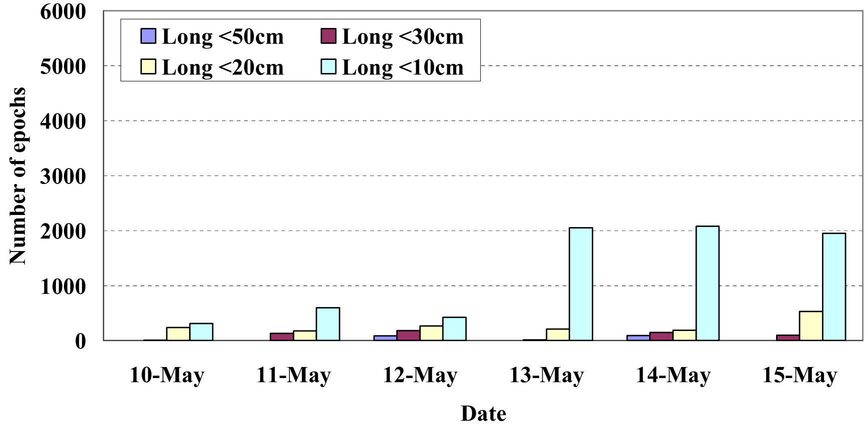
Ambiguity Convergence Analysis



Latitude Error

(Site: Calgary Precise clock rate: 30 sec Observation data rate: 1 sec)

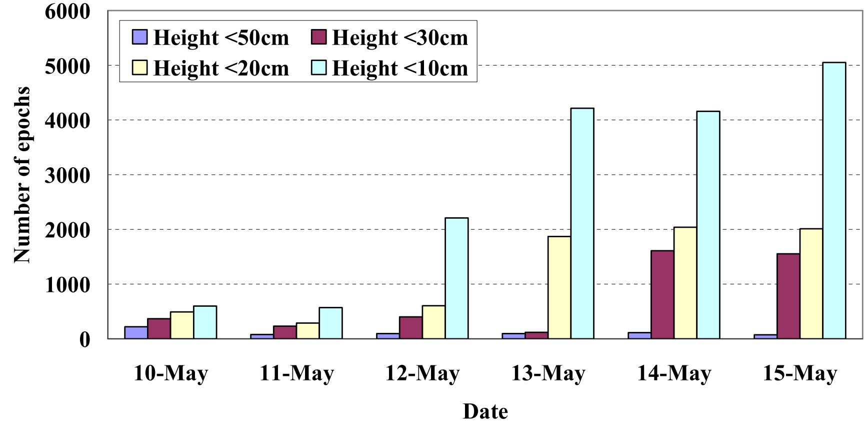
Ambiguity Convergence Analysis



Longitude Error

(Site: Calgary Precise clock rate: 30 sec Observation data rate: 1 sec)

Ambiguity Convergence Analysis



Height Error

(Site: Calgary Precise clock rate: 30 sec Observation data rate: 1 sec)

Single Frequency PPP

- **Pseudorange and carrier phase observables**

$$P(L1) = \rho + c(dt - dT) + d_{orb} + d_{trop} + d_{ion/L1} + d_{mult/P(L1)} + \varepsilon(P(L1))$$

$$\Phi(L1) = \rho + c(dt - dT) + d_{orb} + d_{trop} - d_{ion/Li} + \lambda_1 N_1 + \lambda_1(\phi_r(t_0, Li) - \phi_s(t_0, Li)) + d_{mult/\Phi(L1)} + \varepsilon(\Phi(L1))$$

- **Ionosphere treatment is the key**
  - Formulate ionosphere-free observable by code and phase combination
  - Correcting ionosphere by models (Klobuchar model and Global Ionospheric Model (GIM))
  - Estimating ionosphere in PPP
- **Usefulness of single-frequency PPP**
  - Low-cost PPP positioning system
  - Sub-decimeter static and sub-meter kinematic positioning accuracy

Ionosphere-free Combination

- **Advantage**
  - Remove first order ionospheric effects completely
  - Do not need any ionospheric products or models
- **Limitations**
  - Phase ambiguity is introduced
  - Observable noise level is dominated by code measurements
  - Very long convergence time
  - Post-processing with long time cycle-slip free observations
- **Accuracy**
  - Centimeter static and sub-meter kinematic positioning accuracy is achievable after ambiguity convergence

Klobuchar Model

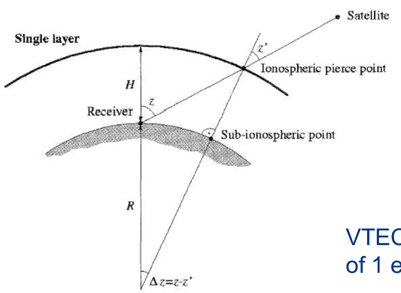
- **Advantage**
  - Easy to implement
  - Model coefficients are available in real-time
- **Limitations**
  - can remove only about 50% ionospheric effects
- **Accuracy**
  - 2 ~ 20 meters
- **Post-fit Klobuchar-Style ionospheric coefficients**
  - IGS Data Analysis Centre CODE has estimated Klobuchar-style ionospheric coefficients that best fit its ionospheric products
  - Studies have confirmed that the post-fit coefficients perform better than the coefficients broadcast by the GPS satellites
  - About 5 days and 1 day latency for final and rapid post-fit coefficients

Global Ionospheric Model (1/2)

- **Advantage**
  - Easy to implement
  - High accuracy (up to 2 TECU at grid points)
- **Limitations**
  - Low spatial resolution: 5 degree (longitude) x 2.5 degree (latitude)
  - Low temporal resolution: 2 hours interval
  - Accuracy degrades after interpolation
  - Latency: 11 days
- **Accuracy**
  - Sub-meter during ionosphere quiet period
- **Rapid ionosphere products**
  - Rapid products have been available at several analysis centers
  - IGS-combined rapid product is also generated
  - IGS rapid ionosphere product is available with latency less than 1 day

Global Ionospheric Model (2/2)

Single Layer Model (Schaer, 1999)



$$TEC = F(z) \bullet VTEC$$

$$F(z) = \frac{1}{\cos z'} = \frac{1}{\sqrt{1 - \sin^2 z'}}$$

$$\sin z' = \frac{R}{R+H} \sin z$$

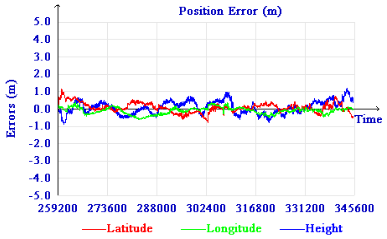
VTEC of each satellite at sub-ionospheric point of 1 epoch (2003/12/3 from S1)

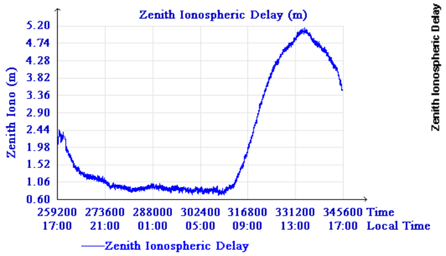
PRN	1	2	3	8	10	13	27	28	29	30
Azimuth (deg)	153	61	60	286	290	166	34	229	318	111
Elevation (deg)	11	19	32	54	25	39	87	23	12	45
VTEC @ sub-ionospheric-point	25.5	22.9	22.6	20.8	18.3	23.3	22.0	21.1	14.3	23.4

Ionosphere Estimated in PPP

- Estimate 1 zenith ionospheric delay at each epoch, map it to slant delay for each satellite using mapping functions
  - Zenith ionospheric delays for different satellites at an ionospheric pierce point are significantly different
  - About 5 meters accuracy has been demonstrated
- Ionospheric horizontal gradients have been demonstrated by researchers for years
  - The general equator toward increase of total electron content (TEC) in mid-latitudes during daytime
  - The west to east increase of TEC in morning in all seasons
  - The east to west increase in the afternoon in winter
- Ionospheric horizontal gradients can be estimated along with zenith delays

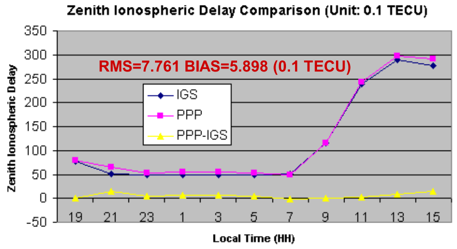
Single Frequency Kinematic Positioning – Ionosphere estimated





Accuracy statistics (Unit: m)

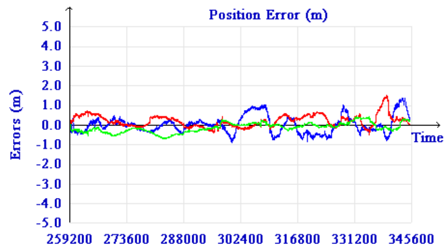
	Lat	Lon	Hgt
BIAS	0.027	-0.108	0.085
RMS	0.277	0.223	0.371
STD	0.268	0.195	0.361



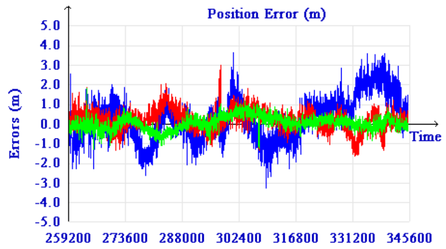
Date : 2003-12-3
Kp index : 7-
Latitude : 51° 4' N Longitude : 114° 7' W
Receiver : Javad (Legacy)
Data interval : 10s
Orbits & Clocks : JPL real-time corrections
Dynamics: Static

Single Frequency Kinematic Positioning – GIM

Code + Phase



Code only



Accuracy statistics (Unit: m)

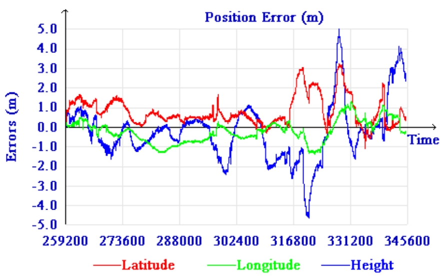
	Lat	Lon	Hgt
BIAS	0.208	-0.131	0.012
RMS	0.366	0.274	0.443
STD	0.302	0.241	0.443

Accuracy statistics (Unit: m)

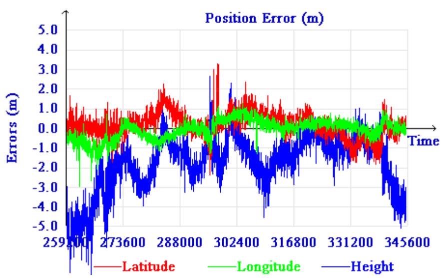
	Lat	Lon	Hgt
BIAS	0.211	0.052	0.249
RMS	0.532	0.340	1.128
STD	0.488	0.336	1.100

Single Frequency Kinematic Positioning – Klobuchar

Code + Phase



Code only



Accuracy statistics (Unit: m)

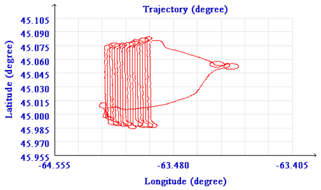
	Lat	Lon	Hgt
BIAS	0.801	-0.221	0.246
RMS	1.088	0.611	1.504
STD	0.735	0.569	1.484

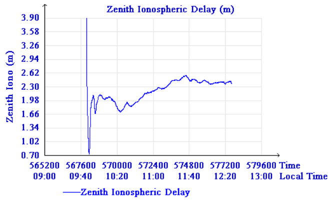
Accuracy statistics (Unit: m)

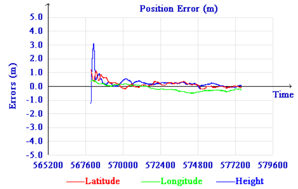
	Lat	Lon	Hgt
BIAS	0.176	-0.057	-1.747
RMS	0.637	0.487	2.262
STD	0.612	0.484	1.437

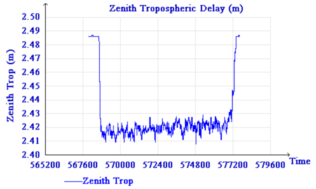
- **Performance is correlated with ionospheric conditions**
- **For each model, positioning accuracy is higher in ionosphere quiet days than in ionosphere disturbed days, and performance is better in mid-latitude and high-latitude region than equatorial region**
- **At mid- and high-latitude region, even the worst model can provide sub-meter level accuracy in ionosphere quiet days.**
- **At equatorial region, even the best model can just achieve about 1 meter level accuracy in ionosphere disturbed days**
- **For the same station, ionosphere estimation model and global ionospheric model are better than Klobuchar model**
- **Ionosphere estimation model and global ionospheric model provide comparable accuracy at middle latitude region. But the latter is slightly more accurate than the former at high latitude region, and the former is much better than the latter at equator region**
- **Carrier phase observations can always improve positioning accuracy when using global ionospheric model and precise GPS orbit and clock products, but they can only improve accuracy in middle and high latitude region for Klobuchar model**
- **The demonstrated positioning accuracy using Klobuchar model or with ionosphere estimated in PPP is obtainable in real-time while it is obtainable only with latency of about 11 days using the global ionospheric model**

Single Frequency Kinematic Positioning – Ionosphere estimated









Accuracy statistics (Unit: m)

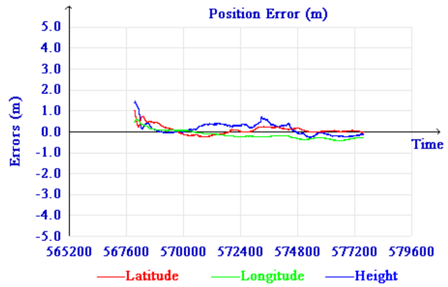
	Lat	Lon	Hgt
BIAS	0.064	-0.189	0.165
RMS	0.154	0.271	0.217
STD	0.140	0.195	0.141

Trajectory, position error, zenith ionospheric delay (estimated) and zenith tropospheric delay (modeled)

Date : 2004-08-28
Kp index : 14-
Latitude : 45° 3' N
Longitude : 63° 27' W
Receiver : NovAtel (Black Diamond)
Data interval : 1s
Orbits & Clocks : JPL real-time corrections
Dynamics: Aircraft

Single Frequency Kinematic Positioning – GIM

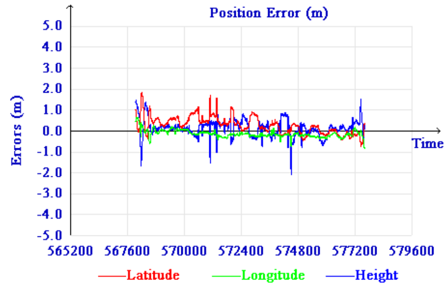
Code + Phase



Accuracy statistics (Unit: m)

	Lat	Lon	Hgt
BIAS	**0.028**	**-0.197**	**0.107**
RMS	**0.131**	**0.243**	**0.262**
STD	**0.128**	**0.143**	**0.239**

Code only

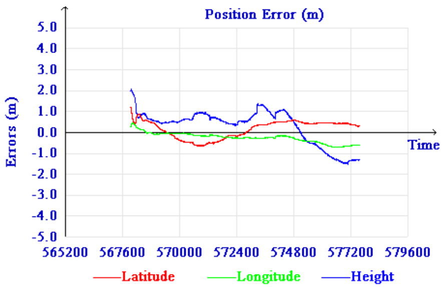


Accuracy statistics (Unit: m)

	Lat	Lon	Hgt
BIAS	**0.261**	**-0.139**	**0.123**
RMS	**0.482**	**0.222**	**0.448**
STD	**0.405**	**0.173**	**0.431**

Single Frequency Kinematic Positioning – Klobuchar

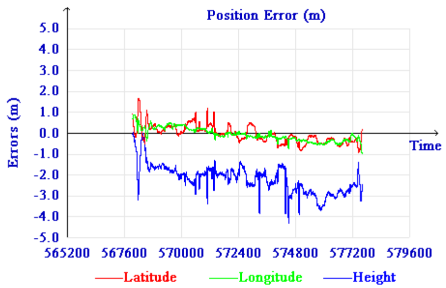
Code + Phase



Accuracy statistics (Unit: m)

	Lat	Lon	Hgt
BIAS	**0.132**	**-0.278**	**0.230**
RMS	**0.418**	**0.347**	**0.840**
STD	**0.396**	**0.208**	**0.808**

Code only



Accuracy statistics (Unit: m)

	Lat	Lon	Hgt
BIAS	**-0.051**	**-0.069**	**-2.215**
RMS	**0.419**	**0.322**	**2.317**
STD	**0.416**	**0.315**	**0.681**

- **Sub-meter level accuracy is obtained using single-frequency GPS observation and JPL real-time corrections**
- **Ionospheric delay estimated in PPP can provide real-time positioning accuracy comparable with global ionospheric model, which uses the IGS Final ionospheric products with about 11 days latency currently**
- **Carrier phase observations can always improve positioning accuracy when using global ionospheric model and precise GPS orbit and clock products, but they can only improve accuracy in middle and high latitude region for Klobuchar model**
- **Single frequency GPS users are looking forward to high temporal and spatial resolution ionospheric TEC grids with short latency**
- **Real-time kinematic tests with low-cost single-frequency GPS receivers should be carried out in the near future**

Section 5

Ambiguity Resolution in PPP

- PPP ambiguity resolution methods
- Comparison and performance
- PPP AR applications
- PPP AR correction messages
- Looking ahead…

PPP ambiguity resolution methods (1/5)

- GPS raw code and phase observations
  - Observable-dependent receiver/satellite code/phase biases
  - Integer ambiguity
  - All receiver/satellite code/phase biases must be isolated or calibrated from the ambiguity parameters. Otherwise, they will be absorbed by the integer ambiguity and thus prevent the PPP integer ambiguity resolution.

$$
\begin{cases}
P_1 = \rho + d_{orb} + c(dt^r - dt^s) + T + I_1 + b^r_{P1} - b^s_{P1} + n_{P1} \\
P_2 = \rho + d_{orb} + c(dt^r - dt^s) + T + I_2 + b^r_{P2} - b^s_{P2} + n_{P2} \\
L_1 = \rho + d_{orb} + c(dt^r - dt^s) + T - I_1 - \lambda_1 N_1 + b^r_{L1} - b^s_{L1} + n_{L1} \\
L_2 = \rho + d_{orb} + c(dt^r - dt^s) + T - I_2 - \lambda_2 N_2 + b^r_{L2} - b^s_{L2} + n_{L2}
\end{cases}
$$

Int amb — rcv/sat code/phase biases

PPP ambiguity resolution methods (2/5)

Sat code clock

$$
\begin{cases}
P_3 = \rho + d_{orb} + c((dt^r + b^r_{P3}) - (dt^s + b^s_{P3})) + T + \varepsilon_{P3} \\
L_3 = \rho + d_{orb} + c((dt^r + b^r_{P3}) - (dt^s + b^s_{P3})) + T - \lambda_3 N_3 + (b^r_{L3} - b^r_{P3}) - (b^s_{L3} - b^s_{P3}) + \varepsilon_{L3}
\end{cases}
$$

Real-valued amb

- Un-difference conventional PPP model
  - *One corrections* for each satellite
    - Satellite code clock
  - One receive code clock
  - The estimated N3 ambiguities are real-valued
    - Contaminated by the receiver and satellite code and phase biases.

PPP ambiguity resolution methods (3/5)

Decoupled sat clocks

$$
\begin{cases}
P_3 = \rho + d_{orb} + c(dt^r_{P3} - dt^s_{P3}) + T + \varepsilon_{P3} \\
L_3 = \rho + d_{orb} + c(dt^r_{L3} - dt^s_{L3}) + T - \lambda_3(17N_1 + 60N_4) + \varepsilon_{L3} \\
A_4 = (b^r_{A4} - b^s_{A4}) - \lambda_4 N_4 + \varepsilon_{A4}
\end{cases}
$$

Integer amb

- Un-difference decoupled clock model
  - *Three corrections* for each satellite
    - Satellite decoupled code clock
    - Satellite decoupled phase clock
    - Satellite MW bias
  - Three receive clock parameter: code clock, phase clock and A4 bias
  - The estimated N1 and N4 ambiguities are integer-valued.

PPP ambiguity resolution methods (4/5)

WSB — *Epoch-averaging to remove rcv N4 bias*

$$A_{4\_urs} + b^s_{A4} = -\lambda_4 N_{4\_urs} + b^r_{A4\_urs} \;\longrightarrow\; -\lambda_4 N_{4\_urs}$$

Integer N4 amb

Sat code clock

$$\begin{cases} P_3 = \rho + d_{orb} + c(dt^r_{P3} - dt^s_{P3}) + T + \varepsilon_{P3} \\ L_3 - 60\lambda_3 N_4 = \rho + d_{orb} + c(dt^r_{L3} - dt^s_{L3}) + T - 17\lambda_3 N_1 + \varepsilon_{L3} \end{cases}$$

Sat integer phase clock — Integer N1 amb

- **Un-difference integer phase clock model**
  - ***Three corrections*** **for each satellite**
    - **Satellite code clock**
    - **Satellite integer phase clock**
    - **Wide-lane sat bias (WSB)**
  - **Two receive clock parameter: code clock, phase clock**
  - **Epoch-averaging required to remove receiver N4 bias**
  - **The estimated N1 and N4 ambiguities are integer-valued**

PPP ambiguity resolution methods (5/5)

Sat code clock

$$\begin{cases} P_3 = \rho + d_{orb} + c((dt^r + b^r_{P3}) - (dt^s + b^s_{P3})) + T + \varepsilon_{P3} \\ L_3 = \rho + d_{orb} + c((dt^r + b^r_{P3}) - (dt^s + b^s_{P3})) + T - \lambda_3 N_3 + (b^r_{L3} - b^r_{P3}) - (b^s_{L3} - b^s_{P3}) + \varepsilon_{L3} \end{cases}$$

Single-difference between sat pair (i,j)

Amb decomposition

$$\lambda_3 N_3 = \lambda_3 (17 N_1 + 60 N_4)$$

$$\Delta A_4{}^{j,i}{}_{\_usr} + \Delta b^{s}_{A4}{}^{j,i} = -\lambda_4 \Delta N_4{}^{j,i}{}_{\_usr} \qquad \Delta A_1{}^{j,i}{}_{\_usr} + \Delta b^{s}_{A1}{}^{j,i} = -17\lambda_3 \Delta N_1{}^{j,i}{}_{\_usr}$$

N4 FCB — Integer N4 amb — N1 FCB — Integer N1 amb

- **Single-difference between satellites method**
  - ***Three corrections*** **for each satellite**
    - **Satellite code clock**
    - **N4 FCB**
    - **N1 FCB**
  - **One receiver code clock**
    - **The corrected N1 and N4 ambiguities are integer-valued.**

Comparison and performance (1/8)

Method comparison for user implementations

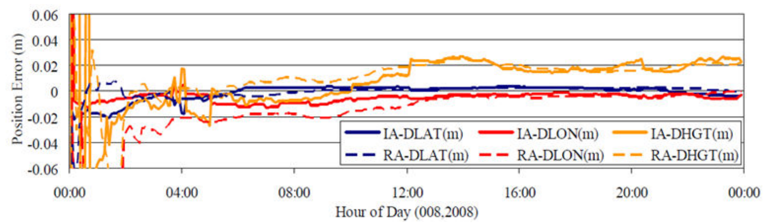
	Conventional PPP	Decoupled clocks	Integer phase clock	Single-difference between satellites
Satellite clocks	Code clock (1)	Code clock (1) Phase clock (2) A4 bias (3)	Code clock (1) Phase clock (2)	Code clock (1)
Receiver clocks	Code clock	Code clock Phase clock A4 bias	Code clock Phase clock	Code clock
Estimated ambiguities	Real-valued N4 Real-valued N1	Integer-valued N4 Integer-valued N1	Real-valued N4 Integer-valued N1	Real-valued N4 Real-valued N1
N4 ambiguity correction	N/A	N/A	WSB (3)	Wide-lane FCB (2)
N1 ambiguity correction	N/A	N/A	N/A	Narrow-lane FCB (3)
Calibrated ambiguities	Real-valued N4 Real-valued N1	Integer-valued N4 Integer-valued N1	Integer-valued N4 Integer-valued N1	Integer-valued N4 Integer-valued N1

Superscript (1) (2) (3) represents the necessary corrections for various PPP models

Comparison and performance (2/8)

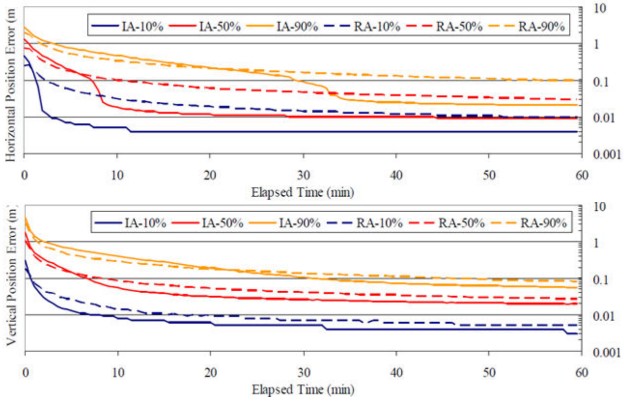
- **Decoupled clock model – daily solution**
  - **Most significant improvement identified in Longitude.**
  - **Less significant improvement identified in Height.**

RMS (cm)	Lat.	Lon.	Hgt.
Conventional	0.6	0.8	1.4
Float decoupled-clock	0.4	0.6	1.4
Fixed decoupled-clock	0.5	0.4	1.4



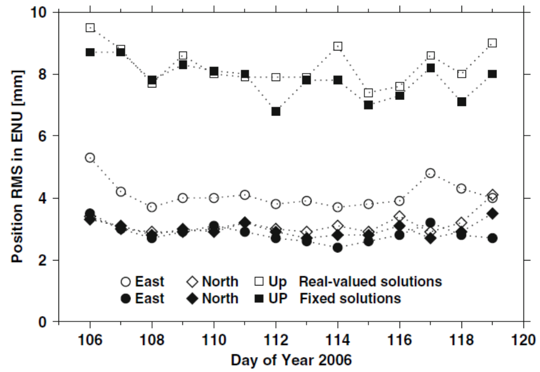
Comparison and performance (3/8)

- Decoupled clock model – hourly solution
  - Same phenomenon as for daily solution – significant horizontal improvement and insignificant in vertical.
  - 90% of hourly solutions reach < 2 cm horizontal accuracy.
  - Vertical improvement is less significant.



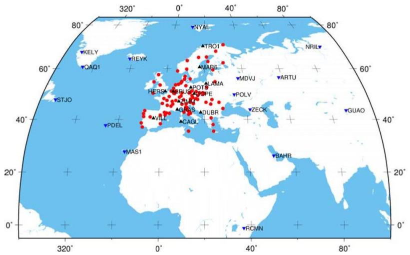
Comparison and performance (4/8)

- Single-differenced between satellites method – daily solution
  - (2.8, 3.0, 7.8) mm fixed versus (4.1, 3.1, 8.3) mm float accuracy in E/N/U;
  - ~ 30% improvement identified in the east component.



Comparison and performance (5/8)

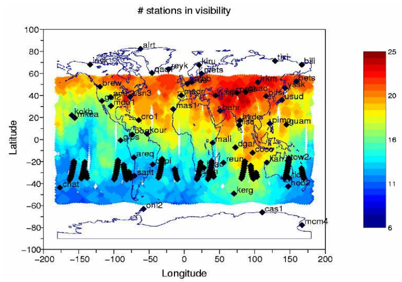
- Single-differenced between satellites method – hourly solution
  - A network of about 80 EUREF stations used to determine FCBs.
  - Performance analysis conducte
    - 12 inside-network stations
    - 15 outside-network stations

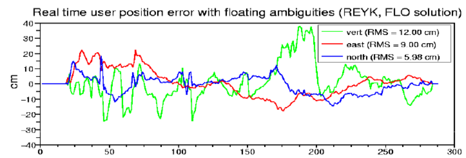


Station location	Float ambiguity (cm)			Fixed ambiguity (cm)		
	East	North	Up	East	North	Up
Inside	3.8	1.5	2.8	0.5	0.5	1.4
Outside	3.7	1.5	3.2	0.6	0.6	2.0

Comparison and performance (6/8)

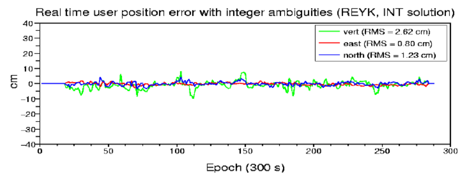
- A network of ~ 50 IGS stations evenly distributed around the world to generate real-time satellite wide-land bias and integer phase





Float solution

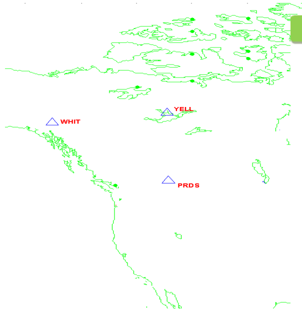
RMS (cm)	N	E	U
Float solution	5.98	9.00	12.00
Fixed solution	1.23	0.80	2.62



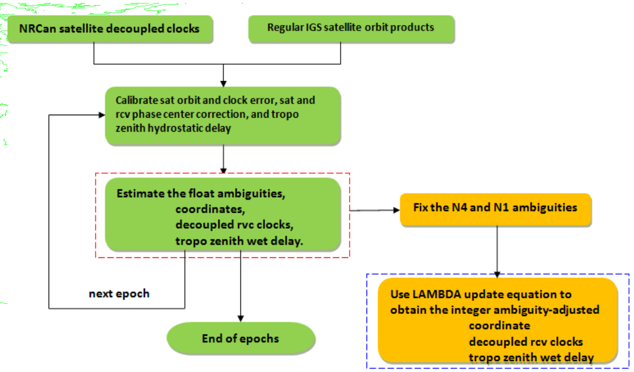
Fixed solution

Comparison and performance (7/8)

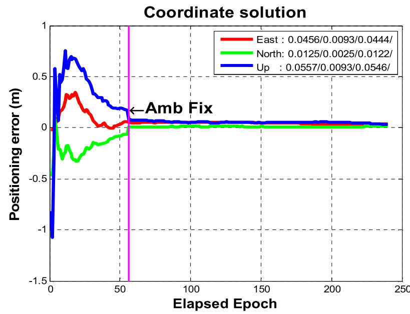
- **PPP-AR at University of Calgary as the first user confirmation**
  - All deployed station equipped with met equipments
  - Troposphere delay estimation for each epoch

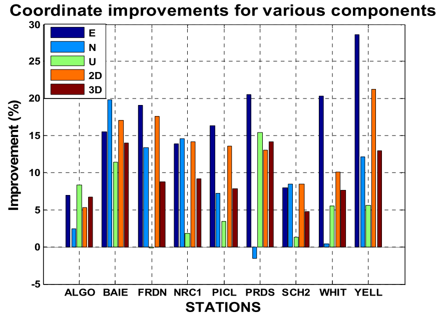


Hourly GPS obs with real met data



Comparison and performance (8/8)





RMS (cm)	N	E	U
Float solution	4.51	3.60	5.75
Fixed solution	3.75	3.27	5.47

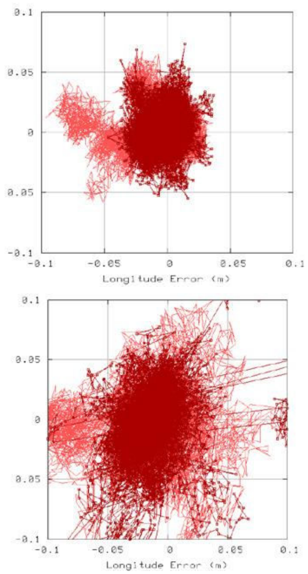
	< 30 min	< 45 min	< 60 min
average	67.57	82.88	89.19

- Accuracy improvements
  - 4.51/3.60/5.75 -> 3.75/3.28/5.47 cm
  - ***16.85/8.85/4.91%*** improve in E/N/U
- Convergence time
  - ***89.19%*** fixed within one hour

Applications (1/4)

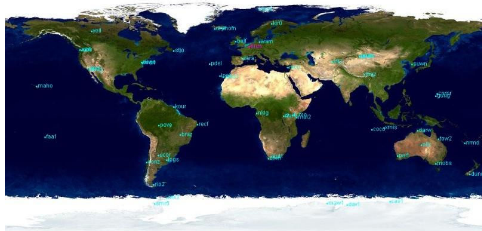
- **Tsunami warning system**

	Float (cm)	Fixed (cm)
Horizontal	4.6	3.3

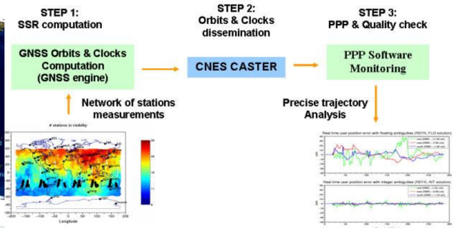


Applications (2/4)

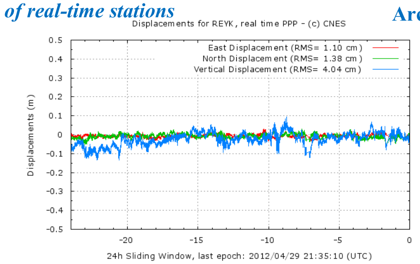
- **Real-time PPP ambiguity resolution demonstrator**



Network of real-time stations



Architecture of the demonstrator

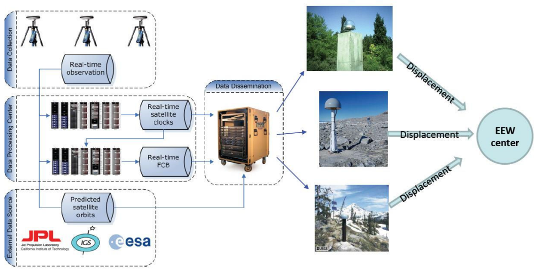


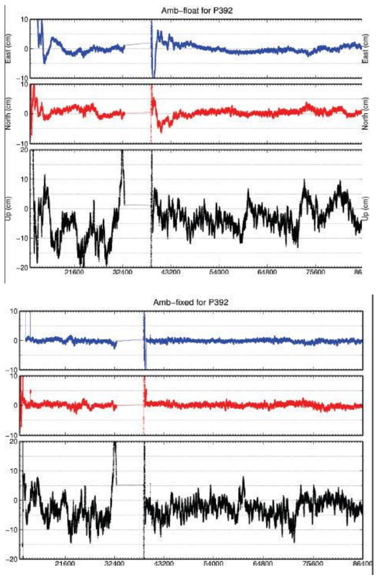
Real-time PPP monitoring at REYK

Applications (3/4)

Earthquake early warning (EEW) system

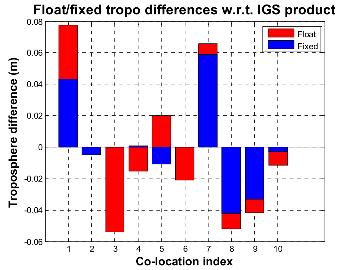
- 50 stations to estimate clocks and FCBs
- All stations far from western US coast
- Ambiguity resolution attempt at every epoch

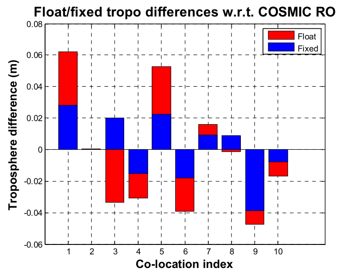




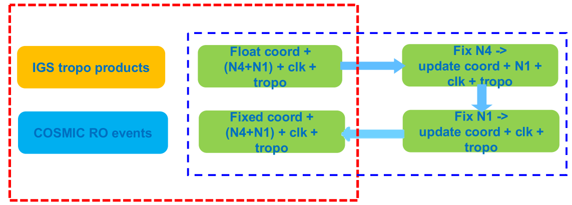
Material provided by Dr. Jianghui Geng

Applications (4/4)





Ambiguity-resolved PPP troposphere estimation



	Float- RO	Fixed- RO	Float-IGS	Fixed-IGS
Mean (cm)	-0.38	0.09	-0.37	0.10
RMS(cm)	3.79	2.11	4.57	3.05

- Mean bias
  - -0.37 to 0.10 cm w.r.t. IGS products;
  - -0.38 to 0.09 cm w.r.t. COSMIC RO observations.
- RMS
  - 4.57 to 3.05 cm ***(33.3%)*** w.r.t. IGS products;
  - 3.79 to 2.11 cm ***(44.3%)*** w.r.t. COSMIC RO observations.

PPP AR correction messages (1/3)

Correction messages from NRCan's decoupled clock model

```
c sat prn01 2008  3  3  0  0   0.0000  0.0001894758704  0.0000000000120  9 i 0027  -8.3101   9.8442 112.0503   0.3290
c sat prn02 2008  3  3  0  0   0.0000  0.0001788439185  0.0000000000122 10 i 0081   1.5611   9.6450  64.3386   0.3239
c sat prn03 2008  3  3  0  0   0.0000  0.0001949590512  0.0000000000127  7 i 0034  -7.6395  10.7281 120.6425   0.3581
c sat prn04 2008  3  3  0  0   0.0000 -0.0000190575649  0.0000000000114 12 i 0094 -20.5793   8.8713 136.3520   0.2997
c sat prn05 2008  3  3  0  0   0.0000  0.0007089269849  0.0000000000122 14 i 0034  -0.2409   9.9648  57.5987   0.3338
...
c sat prn28 2008  3  3  0  0   0.0000 -0.0000160358652  0.0000000000100  9 i 0012  -9.3186   7.9211  97.7361   0.2641
c sat prn29 2008  3  3  0  0   0.0000 -0.0000533654634  0.0000000000133  8 i 0038  -1.9133  11.1108  62.0793   0.3739
c sat prn30 2008  3  3  0  0   0.0000  0.0000659907946  0.0000000000126 12 i 0008  -3.5860  10.3909  54.3659   0.3482
c sat prn31 2008  3  3  0  0   0.0000 -0.0000113951967  0.0000000000126 13 i 0065   4.1407  10.5599  50.4907   0.3530
c sat prn32 2008  3  3  0  0   0.0000  0.0002412456901  0.0000000000122 11 i 0032  -8.8941  10.0071  83.1661   0.3349
```

Sat decoupled phase clock

Sat decoupled code bias

Sat A4 bias

PPP AR correction messages (2/3)

Correction messages from CNES's integer phase clock model

```
     2.00           C                                       RINEX VERSION / TYPE
SparseToClk         CNES/POD SERVICE    20120229 000000     PGM / RUN BY / DATE
     1    AS                                                # / TYPES OF DATA
WIDELANE SATELLITE FRACTIONAL BIASES (2012/02/29)           COMMENT
 G01   0.3091                                               COMMENT
 G02  -0.1180                                               COMMENT
 G03  -0.2785                                               COMMENT
 G04  -0.0336                                               COMMENT
 G05  -0.4082                                               COMMENT
...
 G29   0.0698                                               COMMENT
 G30   0.4429                                               COMMENT
 G31   0.0514                                               COMMENT
 G32  -0.0036                                               COMMENT
                                                            END OF HEADER
AS G02  2012 02 29 00 00  0.000000  2   3.807928330205E-04  8.702687243720E-12
AS G03  2012 02 29 00 00  0.000000  2   1.572428449818E-05  1.084416873489E-11
AS G04  2012 02 29 00 00  0.000000  2   1.080611314111E-04  9.413178766492E-12
AS G05  2012 02 29 00 00  0.000000  2  -2.882632571097E-04  1.002693670166E-11
...
AS G29  2012 02 29 00 00  0.000000  2   2.890386755493E-04  7.318396248647E-12
AS G30  2012 02 29 00 00  0.000000  2  -6.293203646904E-05  7.321731889599E-12
AS G31  2012 02 29 00 00  0.000000  2   2.136858639719E-04  7.595254447662E-12
AS G32  2012 02 29 00 00  0.000000  2  -4.139220193458E-04  8.669330834200E-12

 *  2012 02 29 00 00  0.00000000
 PG01 -21076.971896 -13797.445017  -8459.691722 999999.999999
 PG02   -843.865600  16139.820041  21234.981524    380.791758
 PG03   5674.847371 -21885.823086 -14169.821589     15.724878
 PG04 -13539.195561  10708.654549  20023.134665    108.060794
 PG05   1428.064649  26069.842206   4469.086269   -288.263223
 ...
 PG29  20017.627666    -45.956673  17503.357105    289.038143
 PG30   4037.078968 -21939.102471  14055.036684    -62.932663
 PG31  12566.612980 -10401.368937  21209.492283    213.685750
 PG32  -7786.018244 -24479.750301   7869.785234   -413.923805
```

WSB

Sat integer phase clock

Sat code clock

PPP AR correction messages (3/3)

- Correction messages from Nottingham's single-difference between satellites method

```
*  2012 02 29 00 00  0.00000000
PG01 -21076.971896 -13797.445017  -8459.691722 999999.999999
PG02   -843.865600  16139.820041  21234.981524    380.791758
PG03   5674.847371 -21885.823086 -14169.821589     15.724878
PG04 -13539.195561  10708.654549  20023.134665    108.060794
PG05   1428.064649  26069.842206   4469.086269   -288.263223
...
PG29  20017.627666    -45.956673  17503.357105    289.038143
PG30   4037.078968 -21939.102471  14055.036684    -62.932663
PG31  12566.612980 -10401.368937  21209.492283    213.685750
PG32  -7786.018244 -24479.750301   7869.785234   -413.923805
Satellite fractional parts of initial phases                    COMMENT
 2007  9  2  0  0  0.000000                                     TIME OF START
 2007  9  2 23 59 40.000000                                     TIME OF ENDDING
                                                                END OF HEADER
*G01 G02  111   0.093   0.080   0.008  2
 2007  9  2  3  2 30.000000  2007  9  2  4 12  0.000000  13  -0.182   0.001   0.000
 2007  9  2 13 24  0.000000  2007  9  2 17  7  0.000000  97  -0.182   0.002   0.000
*G01 G03  136   0.703   0.081   0.004  3
 2007  9  2  0  0  0.000000  2007  9  2  1  9 30.000000  49   0.285   0.000   0.000
 2007  9  2 17 31 30.000000  2007  9  2 17 54 30.000000   2   0.507   0.000   0.000
 2007  9  2 21 44  0.000000  2007  9  2 23 59 30.000000  81   0.596   0.001   0.000
...
*G01 G31  168   0.527   0.075   0.003  2
 2007  9  2  0  0  0.000000  2007  9  2  4 12  0.000000  83   0.196   0.000   0.000
 2007  9  2 13 55 30.000000  2007  9  2 17 54 30.000000  83   0.228   0.001   0.000
*G01 G32   46   0.769   0.063   0.006  3
 2007  9  2  0  0  0.000000  2007  9  2  2 40 30.000000  18   0.107   0.000   0.000
 2007  9  2 13 14  0.000000  2007  9  2 14 10  0.000000  10   0.150   0.001   0.000
 2007  9  2 22  5 30.000000  2007  9  2 23 59 30.000000  18   0.140   0.002   0.000
```

Sat code clock

Wide-lane FCB

N1 FCB

Looking ahead… (1/5)

- PPP ambiguity resolution is feasible by using one of the three effective methods:
  - Un-differenced decoupled clock model;
  - Un-differenced integer phase clock model;
  - Single-differenced between satellites method.
- All PPP AR methods require three satellite corrections:
  - Decoupled phase clock + decoupled code clock + decoupled A4 bias;
  - Regular code clock + integer phase clock + wide-lane satellite bias;
  - Regular code clock + N1 FCBs + N4 FCBs .
- So far, CNES is regularly providing its PPP ambiguity resolution messages.
  - WSB
  - Integer phase clock

Looking ahead… (2/5)

1. Property and resolution of the satellite/receiver code/phase bias
   - IGS Workshop on GNSS Biases – January 2012
     - GNSS satellite DCB – satellite code bias
     - GNSS satellite UPD – satellite phase bias
     - Receiver tracking loop – receiver code bias
     - Receiver initial phase bias – receiver phase bias
   - *Satellite code biases* investigated by *IGS GNSS bias working group*;
   - *Satellite phase biases* estimated by *PPP-AR service providers*;
   - *Receiver code biases* covered by *receiver manufacturers*;
   - *Receiver phase biases* eliminated or estimated by *PPP-AR users*

Looking ahead… (3/5)

2. Quality assessment of each type of PPP AR corrections
   - All corrections are satellite-dependent;
   - Evaluation of single-differenced AR corrections
     - *N4 FCB;*
     - *N1 FCB.*
   - Evaluation of un-differenced AR corrections
     - *A4 bias / WSB* – similar to SD N4 FCB;
     - *Phase clock versus code clock*
       - IGS satellite clock day-boundary jump
   - Assessment concerning convergence of N4/N1 ambiguity series at the user solution;
   - Receiver code and phase clock comparison

Looking ahead… e (4/5)

3. **PPP-AR model and algorithm consistency**
   - Inconsistency could not maximize user performance of PPP-AR
     – Service providers claim *<1 cm horizontal* and *<2 cm vertical* accuracy.
     – User cannot reach that high accuracy. Normally *~ 3 cm horizontal* and *~ 5 cm vertical*.
   - Model consistency by augmentation systems
     – Global/regional *ionosphere* map (GIM) – IGS SSR working group
     – Regional *troposphere* product
   - Algorithm consistency by PPP-AR correction providers
     – Standardized PPP positioning algorithm
       - P3/L3/A4 (decoupled clock model)
       - A4 -> P3/L3 (integer phase clock / single-difference method)
       - A4 -> L3 (only phase obs to avoid large code noises)

Looking ahead… (5/5)

4. **RTCM implementation of the PPP AR correction messages**
   - Each method requires *3 corrections*
     – *1 clock + 2 biases* (Single-differenced method)
     – *2 clocks + 1 bias* (both un-differenced models)
   - Existing RTCM State Space Representation (SSR) message types
     – Type 1057 - GPS orbit corrections to Broadcast Ephemeris
     – Type *1058* - GPS clock corrections to Broadcast Ephemeris
     – Type *1059* - GPS code biases
     – Type *1060* - Combined orbit and clock corrections to GPS Broadcast Ephemeris
     – Type 1061 – GPS user range accuracy (URA)
     – Type *1062* – High-rate GPS clock corrections to broadcast ephemeris
   - Single-differenced methods
     – *incompatible with current RTCM SSR definitions*
     – *1059/1060/N/A -> bias/clock/bias*
   - Un-differenced models
     – *compatible with current RTCM SSR definitions*
     – *1058/1059/1060 -> clock/bias/clock*
     – *1059/1060/1062 -> bias/clock/clock* (Collins and Lahaye, 2012)

Section 6

Applications and Case Studies

- P3 software description
- Geodetic positioning
- Airborne geo-referencing
- Point RTK positioning
- Atmosphere remote sensing
- Precise time transfer

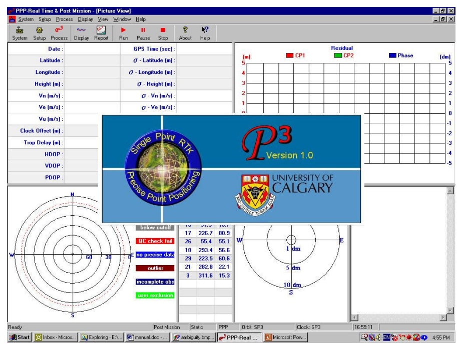
P3 Software Description

Features

- Undifferenced code/carrier processing
- Different observation model implementations
- Precise tropospheric delay and receiver clock estimation
- Static and kinematic positioning
- Forward and backward data processing
- Post-mission PPP using IGS Precise ephemeris and clock products
- Real-time PPP using JPL and NRCan real-time precise orbit/clock products
- Easy-to-use interface
- On-line view of processing results
- Various utilities

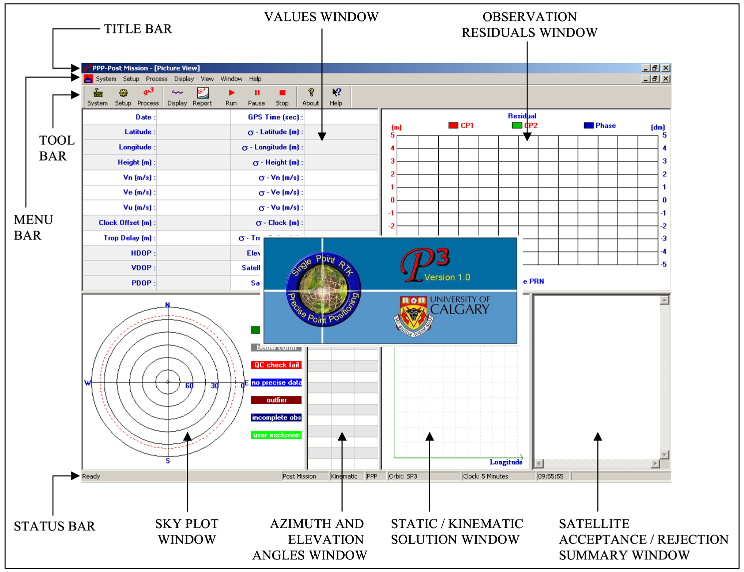
Performance

- Static: mm ~ cm
- Kinematic: cm ~ dm
- Zenith Trop: mm ~ cm
- Receiver clock: sub-ns



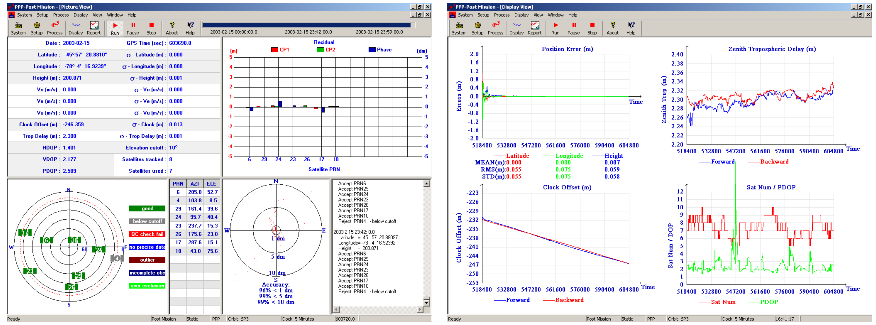
A Window-based Software System for Precise Point Positioning (PPP)

Software Interface

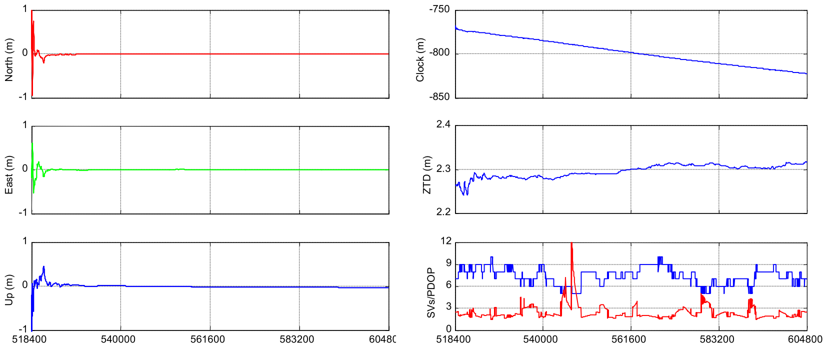


Technology Transfer

is now being licensed to users via University Technology International Inc.



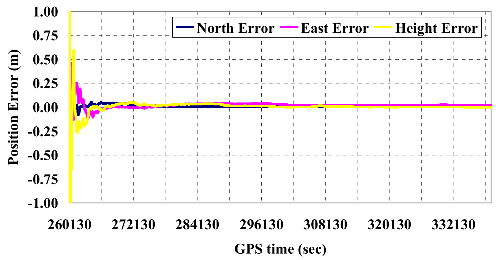
Geodetic Positioning (1/4)



	NORTH	EAST	UP
Bias (m)	0.001	0.002	-0.006
RMS (m)	0.005	0.005	0.019
STD (m)	0.005	0.004	0.018

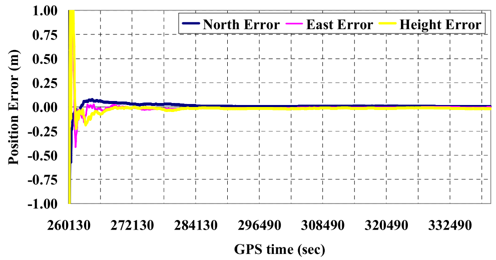
IGS final products, station ALGO

Geodetic Positioning (2/4)



Position Error	RMS(m)	Bias(m)
North	0.013	0.009
East	0.024	0.017
Height	0.017	0.010

CHUR



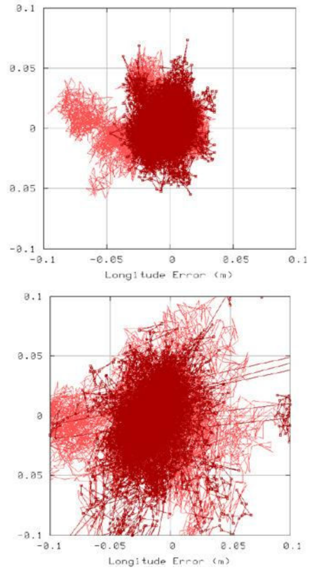
Position Error	RMS(m)	Bias(m)
North	0.021	0.015
East	0.010	-0.008
Height	0.020	-0.015

WHIT

Geodetic Positioning (3/4)

- **Tsunami warning system**

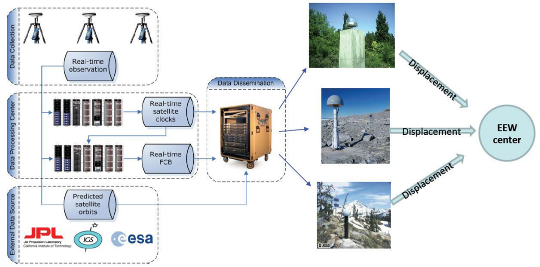
	Amb-float (cm)	Amb-fixed (cm)
Horizontal	4.6	3.3

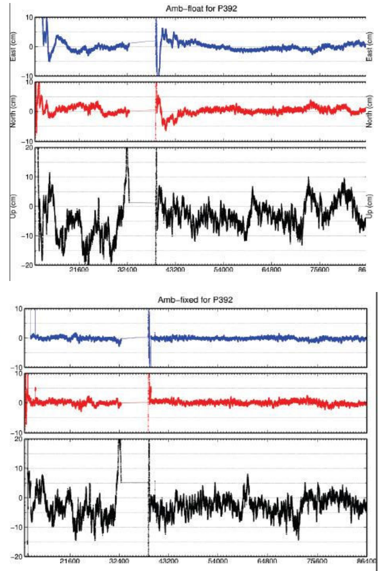


Source: Collins et al. 2009

Geodetic Positioning (4/4)

- **Earthquake early warning (EEW) system**
  - 50 stations to estimate clocks and FCBs
  - All stations far from western US coast
  - Ambiguity resolution attempt at every epoch

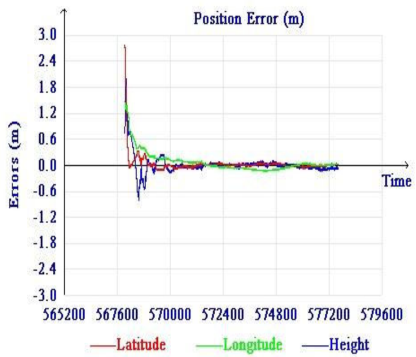


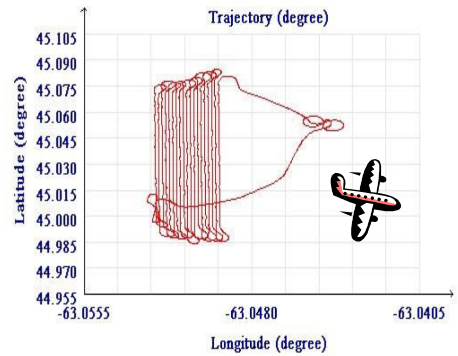


Source: Geng 2012

Airborne Geo-Referencing (1/2)

Airborne Geo-Referencing (2/2)

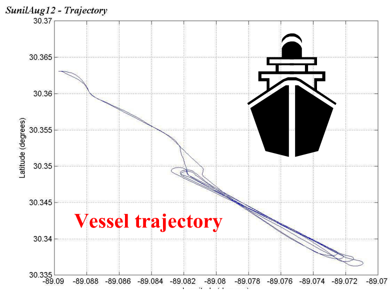


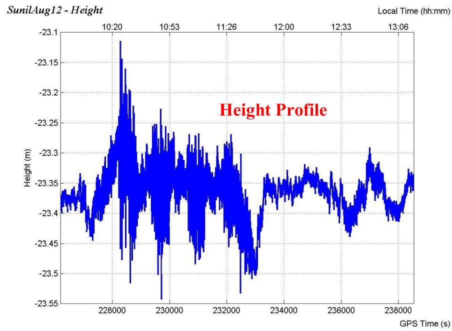


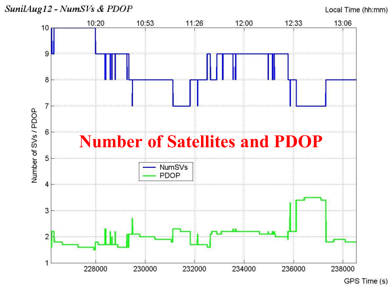
	RMS (m)	BIAS (m)	STD (m)
Latitude	0.025	0.002	0.025
Longitude	0.065	-0.024	0.060
Height	0.047	-0.013	0.045

Point RTK Positioning (1/3)

Point RTK Positioning (2/3)





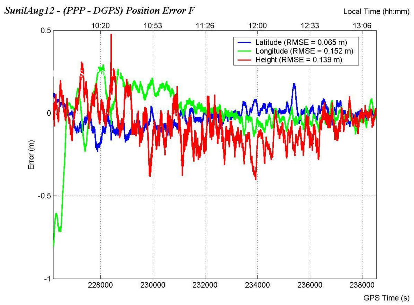


Marine Data

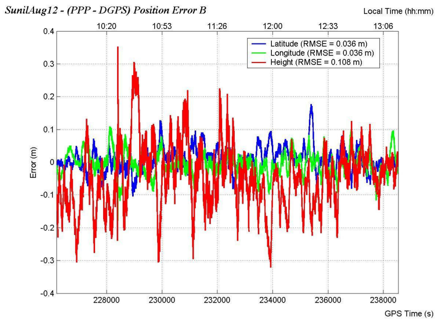
```
Duration: 3:25:45
Data interval (s): 1.0
Satellite Orbit: JPL 15 minute
Satellite Clock: JPL 1.0 second
Processing Interval (s): 1.0
```

Kinematic, JPL real-time orbit and clock data

Point RTK Positioning (3/3)



Position Errors (Forward Processing)

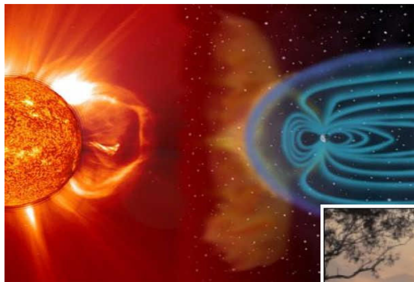


Position Errors (Backward Processing)

	Latitude	Longitude	Height
Mean	0.7	-0.5	-4.9
Std. Dev.	3.6	3.5	9.6
RMS	3.6	3.6	10.8

unit: cm

Atmosphere Remote Sensing (1/8)



Atmosphere Remote Sensing (2/8) –
Atmospheric Water Vapor Estimation Using GPS

- **Precise ZWD estimation requires high precision GPS satellite orbit data**
- **ZWD has to be estimated along with satellite orbit and clock parameters using a network before the emergence of precise orbit and clock data.**
  - ✓ **Data processing procedures are too complex to be feasible for real-time or near real-time ZWD estimation**
- **Availability of precise orbit data can simplify the estimation procedure as the satellite orbit parameter estimation is not necessary.**
  - ✓ **Still need to estimate satellite clock parameters**
- **Double differenced approach has to be employed before the emergence of precise clock data.**
  - ✓ **Provide only relative ZWD estimates for short baselines**
  - ✓ **To recover absolute ZWD from double differenced ZWD solutions**
    - **Require baseline length over 1000km**
    - **Significant constrains on practical use**

    or

    - **Require absolute ZWD from a Water Vapor Radiometer (WVR) at the base station**
    - **WVR is not all weather instrument and is expensive**

Atmosphere Remote Sensing (3/8) –
Atmospheric Water Vapor Estimation Using PPP

- Dual-frequency Un-differenced Observation Combinations

$$P_{IF} = \frac{f_1^2 \cdot P_1 - f_2^2 \cdot P_2}{f_1^2 - f_2^2} = \rho + cdt + d_{trop} + dm_{IF} + \varepsilon(P_{IF})$$

$$\Phi_{IF} = \frac{f_1^2 \cdot \Phi_1 - f_2^2 \cdot \Phi_2}{f_1^2 - f_2^2} = \rho + cdt + d_{trop} + \frac{cf_1 N_1 - cf_2 N_2}{f_1^2 - f_2^2} + \delta m_{IF} + \varepsilon(\Phi_{IF})$$

- Tropospheric Delay Modeling

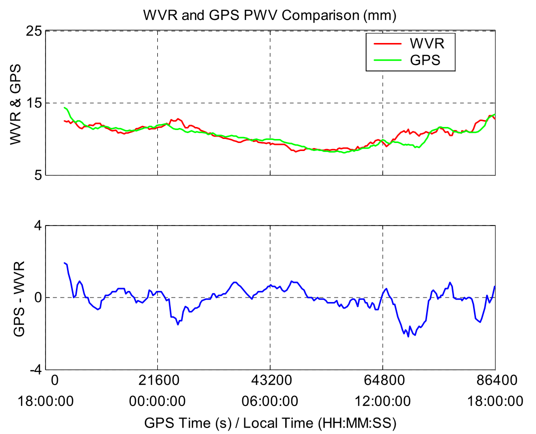
$$d_{trop} = m_h(e)D_{hz} + m_w(e)D_{wz} + m_g(e)[G_N \cos(a) + G_E \sin(a)]$$

$$D_{hz} = \frac{0.0022768\, P_0}{1 - 0.00266 \cos 2\phi - 0.00028\, H}$$

- Mapping Functions
  - ✓ Hydrostatic and wet mapping functions: **Niell Mapping Functions**
  - ✓ Gradient mapping function: $m_g(e) = \frac{1}{\sin(e)\tan(e) + 0.0032}$
- PPP enables:
  - ✓ Absolute Zenith Troposphere Delay (ZTD)
  - ✓ No station constraint and real-time feasible

Atmosphere Remote Sensing (4/8) –
Real-Time ZTD Estimation with JPL Corrections

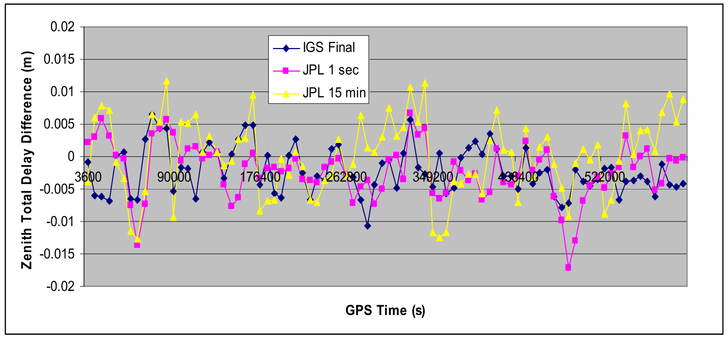
- The ZWDs estimated in the test were converted to PWVs and compared with radiometer measurements.



Accuracy statistics (unit: mm)

Date	RMS	BIAS	STD
Sept. 2	0.93	0.04	0.92
Sept. 3	0.99	0.08	0.99
Sept. 4	1.09	0.30	1.05
Sept. 5	0.77	-0.28	0.72
Sept. 6	0.65	0.01	0.65
Sept. 7	0.64	0.12	0.62
Sept. 8	0.95	-0.28	0.91

Atmosphere Remote Sensing (5/8) –
Tropospheric Delay Estimation Using PPP



Precise Products	RMS (m)	Mean (m)
IGS Final	0.0041	-0.0020
JPL RT 1 sec	0.0052	-0.0022
JPL RT 15 min	0.0059	0.0001

Zenith Total Delay Differences wrt IGS Final Tropospheric Products (station AMC2, GPS Week 1251)

Atmosphere Remote Sensing (6/8) –
Absolute Ionosphere Estimation Using PPP

- PPP Derived Ionosphere Delay Estimates

$$\frac{40.3}{f_1^2}TEC = \frac{f_2^2}{f_1^2 - f_2^2}(\Phi_1 + N_1\lambda_1 - \Phi_2 - N_2\lambda_2 + (1-\frac{f_1^2}{f_2^2})cT'_{GD,r} - (1-\frac{f_1^2}{f_2^2})cT'_{GD,s})$$

$$\frac{40.3}{f_2^2}TEC = \frac{f_1^2}{f_1^2 - f_2^2}(\Phi_1 + N_1\lambda_1 - \Phi_2 - N_2\lambda_2 + (1-\frac{f_1^2}{f_2^2})cT'_{GD,r} - (1-\frac{f_1^2}{f_2^2})cT'_{GD,s})$$

- Ionosphere recovery accuracy

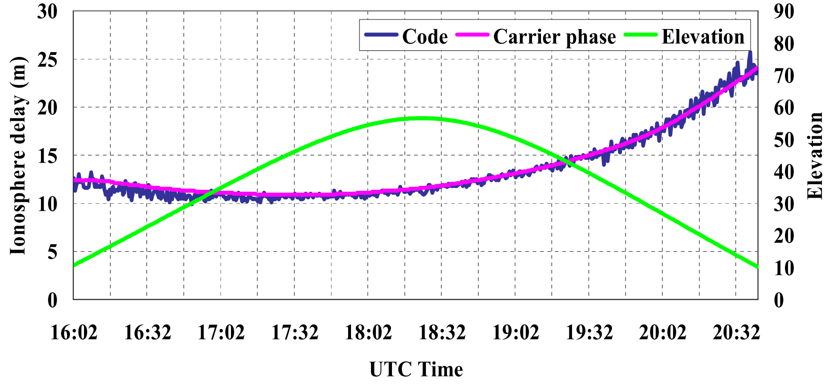
$$\sigma^2_{ion,L1} = (1.54)^2(\sigma^2_{\Phi_1} + \lambda_1^2\sigma^2_{N_1} + \sigma^2_{\Phi_2} + \lambda_2^2\sigma^2_{N_2} + 0.64^2\cdot c^2\cdot\sigma^2_{T'_{GD,r}} + 0.64^2\cdot c^2\cdot\sigma^2_{T'_{GD,s}})$$

$$\sigma^2_{ion,L2} = (2.54)^2(\sigma^2_{\Phi_1} + \lambda_1^2\sigma^2_{N_1} + \sigma^2_{\Phi_2} + \lambda_2^2\sigma^2_{N_2} + 0.64^2\cdot c^2\cdot\sigma^2_{T'_{GD,r}} + 0.64^2\cdot c^2\cdot\sigma^2_{T'_{GD,s}})$$

Decimeter level with converged ambiguity

- Limiting factor
  - ✓ Inter-frequency bias (accuracy is 0.03ns reported normally in the JPL-IONEX file, worse for day-to-day solution).
  - ✓ Ambiguity resolved within a fraction of one cycle

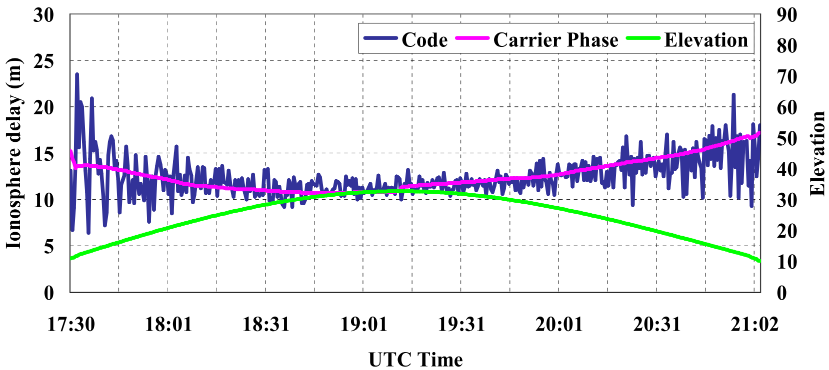
Atmosphere Remote Sensing (7/8) –
Absolute Ionosphere Estimation Using PPP



Ionosphere delay estimation for PRN 29 at WHIT station

Precise product: IGS final (15 min orbit and 5 min clock)
Inter-frequency bias for satellites & receiver: from JPL
Station: IGS WHIT
Data rate: 30 sec

Atmosphere Remote Sensing (8/8) –
Absolute Ionosphere Estimation Using PPP



Ionosphere delay estimation for PRN 29 at PIE1 station

Precise product: IGS final (15 min orbit and 5 min clock)
Inter-frequency bias for satellites & receiver: from JPL
Station: IGS PIE1
Data rate: 30 sec

Precise Time Transfer (1/2)

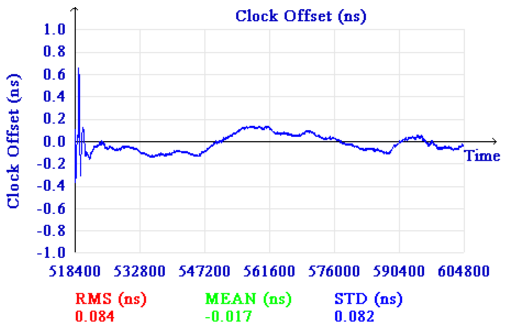
- Dual-frequency Un-differenced Observation Combinations

$$P_{IF} = \frac{f_1^2\cdot P_1 - f_2^2\cdot P_2}{f_1^2 - f_2^2} = \rho + cdT + d_{trop} + dm_{IF} + \varepsilon(P_{IF})$$

$$\Phi_{IF} = \frac{f_1^2\cdot \Phi_1 - f_2^2\cdot \Phi_2}{f_1^2 - f_2^2} = \rho + cdT + d_{trop} + \frac{cf_1N_1 - cf_2N_2}{f_1^2 - f_2^2} + \delta m_{IF} + \varepsilon(\Phi_{IF})$$

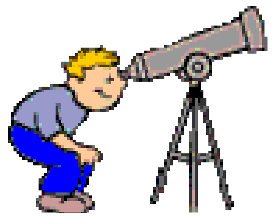
- PPP enables:
  - Receiver clock offset estimation
  - Referred to the reference clock used by the precise clock products

Precise Time Transfer (2/2) – Receiver Clock Estimates

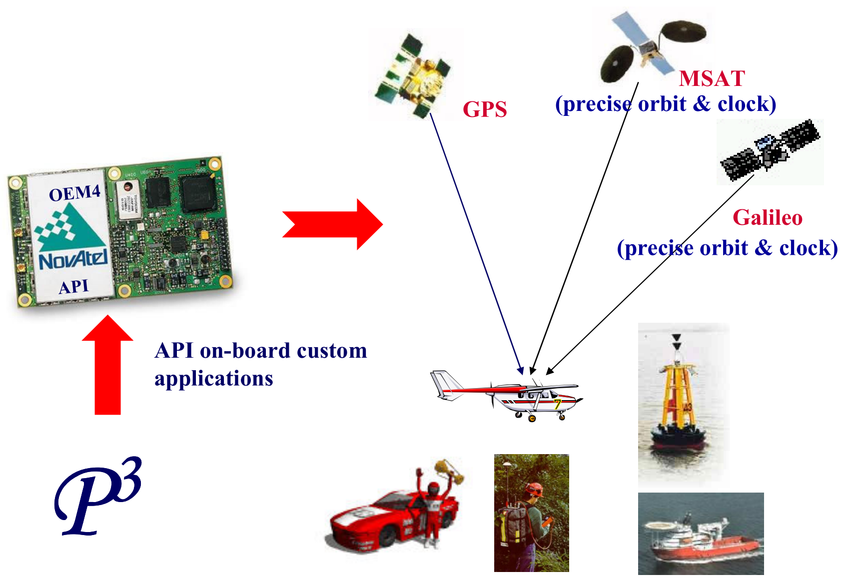


- JPL selects AMC2 clock as reference clock and fixes it to estimate real-time corrections
- June 12, 2004: AMC2 receiver clock offset was estimated as white noise process using JPL real-time corrections to assess the accuracy of receiver clock offset estimation
- The RMS of one day's receiver clock offset estimates is less than 0.1 ns

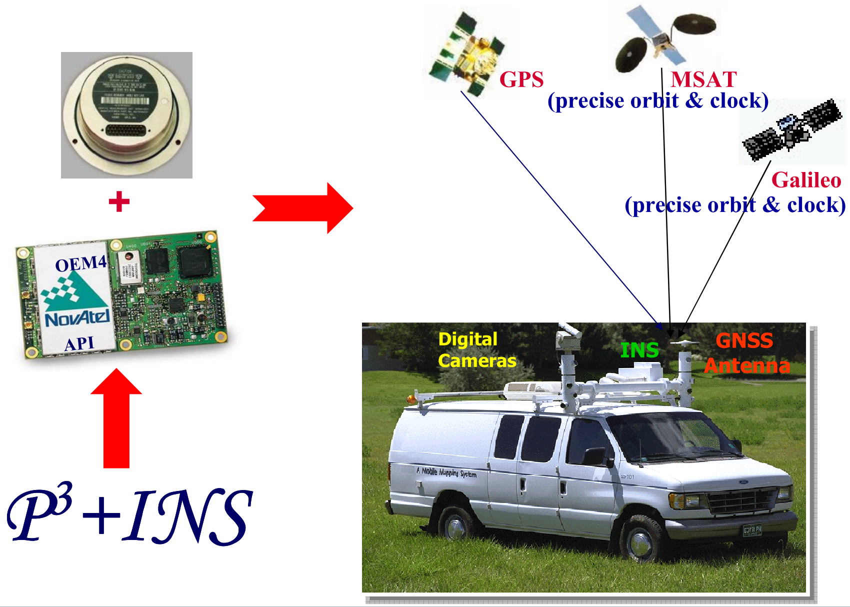
Looking Ahead ….



Point-RTK Receiver System



Point GPS/INS System



RTK in the future

Reference Stations +
Network RTK +
PPP-RTK

RTK Positioning
Anywhere!
Anytime!
Anyway!