# LOCAL MINIMUM DETECTION WITH MATHEMATICAL MORPHOLOGY

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**ABSTRACT**

This paper shows basic concepts of Mathematical Morphology and how to obtain local minimum without a specific threshold. Two equations are developed with the tools of MM to obtain objects that could be seen like a local minimum. The dimension of the structural element defines the minimum region. If structural element is small, more noise could be obtained in the final image. If structural element is large, less noise could be obtained in the final image. This application could be useful to obtain water bodies, roads and other targets with low gray levels of remote sensing medium resolution images.

**Key words**: Mathematical Morphology, Pattern Recognition, Local minimum.

## 1 INTRODUCTION

Mathematical Morphology (MM) is based on a mapping study between complete lattices in terms of some families of simple mappings: dilations, erosions, anti–dilations, anti–erosions and it is a powerful theory to extract image information.

Basic concepts of Mathematical Morphology can be seen in Serra (1988), Barrera (1987), Haralick et al. (1987), Haralick and Shapiro (1991), Banon and Barrera (1994) and Candeias(1997) and Najman and Talbot (2010).

In this paper we will see how to obtain local minimum with the tools of MM.

### 1.1 Basic image operations and transformations

The definitions of this section were extracted from Barrera and Barrera (1994). Let Z be the set of integers. Let $E$ be a rectangle of $Z^2$ and let $K$ be an interval $[0, k]$ of $Z$, with $k > 0$. The collection of functions from $E$ to $K$ will represent the gray–scale images of interest. We denote such a collection by $K^E$ and by $f$, $g$, $f_1$ and $f_2$ generic elements of $K^E$.

Some useful local operations definitions on images are based on the structural properties of the interval $[0, k]$ of the intersection of $f1$ and $f2$, denoted $f1 \wedge f2$, is the function in $K^E$ given by, for any $x$ in $E$,

$$(f_1 \wedge f_2)(x) = \min \{f_1(x), f_2(x)\} \quad (1)$$

The union of $f1$ and $f2$, denoted $f1 \vee f2$, is the function in $K^E$ given by, for any $x$ in $E$,

$$(f_1 \vee f_2)(x) = \max \{f_1(x), f_2(x)\} \quad (2)$$

The binary equality of $f1$ and $f2$, denoted $f1 \equiv f2$, is the function in $K^E$ given by, for any $x$ in $E$. $g$ is a binary result comparative image. If $f1(x) \equiv f2(x)$ then $g(x)= 255$ in $x$ position, otherwise $g(x)=0$.

The two binary operations $\wedge$ and $\vee$ from $K^E$ to $K^E$ to $K^E$ are called, respectively, *intersection* and *union*.

Two important subclasses of dilations and erosions definitions are based on the Abelian group property of ($Z^2$, Z+).

Let $B$ be a subset of $Z^2$, called structural set (or, structural element). We denote by $B_h$ the translate of $B$ by any vector $h$ in $Z^2$, that is,

$$B_h = \{x + h : x \in B\} \quad (3)$$

We denote by $B^c$ the *complement of* $B$, that is,

$$B^c = \{x : x \notin B\} \quad (4)$$

The *dilation of f by B* is the function $\delta_B(f)$ in $K^E$, given by, for any $x$ in $E$,

$$\delta_B(f)(x) = \max \{f(y) : y \in B^t_x \cap E\} \quad (5)$$

The *erosion of f by B* is the function $\varepsilon_B(f)$ in $K^E$, given by, for any $x$ in $E$,

$$\varepsilon_B(f)(x) = \min \{f(y) : y \in B_x \cap E\} \quad (6)$$

### 1.2 Local minimum extraction

Let $f$ be a gray image. Equation (7) obtains the local minimum and homogeneous regions (without gray level variation).

$$f_1 = (\varepsilon_{B1^*}(f) \equiv f) \tag{7}$$

Where B1 is structural element, $n$ is the dimension and $B1^* = nB1$.

$$B1 = \begin{matrix} 1 & 1 & 1 \\ 1 & 1. & 1 \\ 1 & 1 & 1 \end{matrix}$$

Equation (8) obtains the local minimum supposing four directions. The selection depends on of structural element.

$$f_1 = (\varepsilon_{B1^*}(f) \equiv f) \vee (\varepsilon_{B2^*}(f) \equiv f) \vee (\varepsilon_{B3^*}(f) \equiv f) \vee (\varepsilon_{B4^*}(f) \equiv f) \tag{8}$$

Where B$i$ is structural element, $n$ is the dimension and $Bi^* = nBi$ and $i = 1,..., 4$.

$$B1= \begin{matrix} 0 & 1 & 0 \\ 0 & 1. & 0 \\ 0 & 1 & 0 \end{matrix} \quad B2= \begin{matrix} 0 & 0 & 0 \\ 1 & 1. & 1 \\ 0 & 0 & 0 \end{matrix} \quad B3= \begin{matrix} 1 & 0 & 0 \\ 0 & 1. & 0 \\ 0 & 0 & 1 \end{matrix} \quad B4= \begin{matrix} 0 & 0 & 1 \\ 0 & 1. & 0 \\ 1 & 0 & 0 \end{matrix}$$

## 2 RESULT

Figure 1 shows a result applying equation (8) to a TM4 Landsat band ($f$) with structural element showed above (B1, B2, B3 and B4) and with $n = 1$. If $n$ is larger than one, less noise will be seen in Figure 1 (f), (g), (h), (i) and (j).

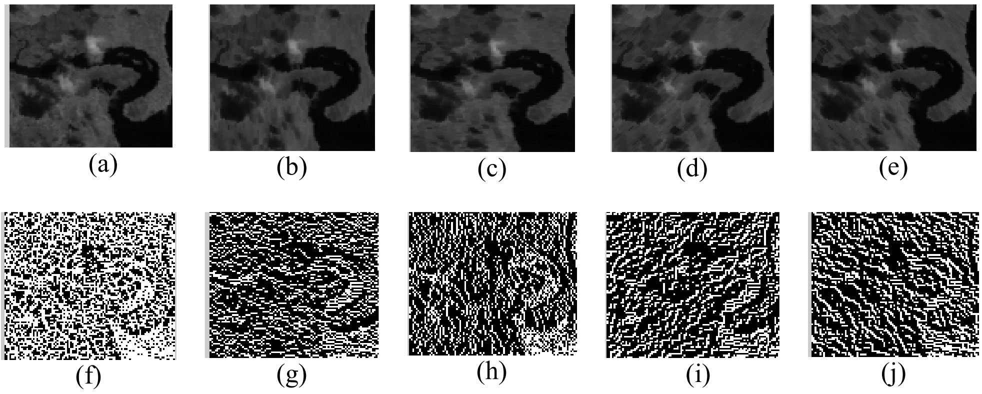


Figure 1 – Result applied to TM4 Landsat band. (a) Original image TM4 (f). (b) $\varepsilon_{B1}(f)$. (c) $\varepsilon_{B2}(f)$. (d) $\varepsilon_{B3}(f)$. (e) $\varepsilon_{B4}(f)$. (f) $(\varepsilon_{B1^*}(f) \equiv f) \vee (\varepsilon_{B2^*}(f) \equiv f) \vee (\varepsilon_{B3^*}(f) \equiv f) \vee (\varepsilon_{B4^*}(f) \equiv f)$. (g) $\varepsilon_{B1}(f) \equiv f$. (h) $\varepsilon_{B2}(f) \equiv f$. (i) $\varepsilon_{B3}(f) \equiv f$. (j) $\varepsilon_{B4}(f) \equiv f$.

## 3 CONCLUSION

This paper shows the basic tools of Mathematical Morphology and how to obtain local minimum without a specific threshold. Equation (7) and (8) could be useful for some application that needs to obtain objects that could be seen like a local minimum.

If structural element is small, more noise could be obtained in the final image. Otherwise, if structural element is large, less noise could be obtained in the final image. This application could be useful to obtain water bodies, roads and other targets with low gray levels of remote sensing medium resolution images.

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