A MULTI-SCALE CURVE MATCHING TECHNIQUE FOR THE ASSESSMENT OF ROAD ALIGNMENTS USING GPS/INS DATA

J. Stratakos, V. Gikas *, K. Fragos

NTUA, School of Rural and Surveying Engineering, National Technical University of Athens,
9 H. Polytechniou St., Zografos, Athens, 15780, Greece – vgikas@central.ntua.gr

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ABSTRACT:

In this paper a methodology and an algorithm is presented for the automated extraction of road alignments using GPS/INS navigational data. Centerline geometry assessment is derived in the form of traditional design elements. The core of the proposed algorithm relies on the use of multi-scale correlation analysis (similarity and affinity transformations) and fundamental curve matching techniques that were suitably adapted to adhere the nature of road alignment data. A mobile mapping vehicle comprising a high accuracy multi-sensor navigational system was used to verify the correctness and level of automation of the algorithm as well as the feasibility of the associated software.

1. INTRODUCTION

Recent advances in land-based mobile mapping systems have resulted an enormous improvement in capturing high quality / low cost field data that can serve for the accurate extraction of road geometry features. Such systems mostly rely on vehicle navigation (GNSS / INS) and optical sensor (image sequence / laser profiler) data that can be processed to acquire road axis, lane line and cross section geometry information [Toth and Grejner Brzezniska, 2002; Wang et al, 2007; Wang et al 2008; Zhiyong et al 2007]. Previous research on road axis geometry extraction mainly concentrates on semi-automated systems that model centerline geometry in the form of a polyline or a line of continuous curvature (e.g. splines) [Cefalo et al,2007; Makanae, 2004]. However, as for the road engineers, centerline geometry is much more useful if presented in terms of traditional design elements, some researchers focused on algorithms to serve this need [Can and Kuscu, 2008; Choi and Sung, 2007; Gikas and Daskalakis, 2008; Karamanou, 2009; Karamanou et al, 2009]. These algorithms, notwithstanding can generally produce satisfactory results, they suffer lack of automation and in certain cases (e.g. noisy data, peculiar geometry) they cannot represent reliably the actual road geometry characteristics.

The core idea behind the existing algorithms relies on azimuth and curvature calculations of the centerline alignment. However, this approach can introduce excessive noise in the computed parameters due to successive derivation operations. This problem could be partially mitigated if a low-pass filter is applied at every derivation step [Stratakos et al, 2009]; howbeit, it can still affect the results as useful information is smeared out due to data filtering - especially, in the clothoid sections. Obviously, in order to overcome this problem an alternative methodology is required that would not depend on curvature computations.

The algorithm proposed in this paper is based on the use of a fundamental curve matching method, known as Direct Curve Matching (DCM) technique [Kohlmayr, 2006; Wolfspon, 1990]. The method originates in the fields of applied mathematics, computational geometry and computer science. Its principle relies on RST (rotation, scaling, translation) computations between two sets of point data so that, the difference in the shape between the curves representing the two data sets comply with certain matching (minimization) criteria. In this study, the first data set corresponds to a subset of the vehicle navigation data reduced on the centerline alignment, whereas the second one corresponds to the reference data of a geometric element (i.e. a circle arc, a clothoid or straight line) of predefined characteristics.

In order to produce reference data for a roadway design element its geometric properties should be taken into account. Thus, the similarity transformation is used to generate a cluster of reference arc circles and clothoids from an initial pair of curves. In contrast, other types of transition curves, such as cubic parabolas, deem necessary the use of affinity transformation. In fact, the similarity and affinity transformations are scale transformations in one and two dimensions respectively. In the first case, the initial and final curves have the same scale ratios in X and Y directions, whereas in the affinity transformation apply different scales [Kohn, 1984]. The conceptual meaning of scale in these transformations as well as in the procedure of reference data preparation is very important throughout this work. Seeing that reference curves of different scales are used for the application of the DCM technique, the proposed methodology resides in the field of multi-scale analysis. Moreover, this analysis may not considered as a trivial one since it ensembles multi-scale transformations with both uniform and non-uniform aspect ratios.

2. MULTI-SCALE CORRELATION

As already stated, scaling plays a fundamental role in the generation of a reference curve and by extension to the implementation of the DCM technique. The similarity transformation of a Euclidean space is defined as a function *f*

* Corresponding author

from the 2D space into itself that multiplies all distances by the same scale factor *r*, so that for any set of points *a* and *b* it applies:

$$d(f(a), f(b)) = r \cdot d(a,b) \quad (1)$$

where, *d(a,b)* is the Euclidean distance between points *a* and *b*.

Hence, in the case of a circle element an initial arc circle is generated and subsequently arc circles of different radii are produced by applying the similarity transformation on the prototype feature. That is, for an arc circle with center coordinates (x_c,y_c) and radius *R* formulated by:

$$(x_c - x)^2 + (y_c - y)^2 = R^2 \quad (2)$$

a similarity transformation of a scale factor *r* will transform its point coordinates *(x,y)* to *(x',y')*, where *x'* and *y'* are defined by equations:

$$x' = r \cdot x \quad (3)$$

$$y' = r \cdot y \quad (4)$$

Substituting *x'* and *y'* coordinates in Eq. 2 it results:

$$(x_c - \frac{x'}{r})^2 + (y_c - \frac{y'}{r})^2 = R^2 \Rightarrow \quad (5)$$

$$(r \cdot x_c - x')^2 + (r \cdot y_c - y')^2 = (r \cdot R)^2 = R'^2 \quad (6)$$

In fact, Eq. 6 provides the parameters (curvature radius and center coordinates) of the transformed circle for a scale factor *r*. Reversely, an arc circle of radius *R'* can be generated from a base circle of radius *R* by multiplying its point coordinates by the scale factor *r* given by:

$$r = \frac{R'}{R} \quad (7)$$

Conclusively, by generating a pool of reference circles it is possible to determine which of them would fit best the sample data. The same working principle applies for the case of clothoid transitions. The parametric equations of a clothoid of a parameter A are given by [Bass, 1984]:

$$x(t) = A \cdot \int_0^t \cos(\frac{\pi \cdot u^2}{2}) \cdot du \quad (8)$$

$$y(t) = A \cdot \int_0^t \sin(\frac{\pi \cdot u^2}{2}) \cdot du \quad (9)$$

From Eq. 8 and 9 it is evident that the clothoid parameter A is simply a constant factor of *du*. Therefore, using Eq. 3 and 4 a similarity transformation will transform *(x,y)* coordinates to *(x',y')* so that, the scale ratio between the initial and the transformed clothoids is:

$$r = \frac{A'}{A} \quad (10)$$

Figure 1 depicts a set of arc circles and a set of clothoid curves of various curvature radii and clothoid parameters generated using a similarity transformation. In road / highway design, in addition to clothoids other transition curves can be used as well. The cubic parabola is such a case, commonly used in railway engineering. This transition curve is defined by:

$$y = K \cdot x^3 \quad (11)$$

where, *K* relates to the curve length and the radius of the adjacent arc circle. In this case, as *y* coordinates relate to *x* coordinates the affinity transformation applies, so that:

$$x' = r_x \cdot x \quad (12)$$

$$y' = r_y \cdot y \quad (13)$$

In order to simplify computations, by setting r_x=1 and substituting Eq. 12 and 13 to Eq. 11 we get:

$$y' = (r_y \cdot K) \cdot x'^3 = K' \cdot x'^3 \Rightarrow \quad (14)$$

$$r_y = \frac{K'}{K}, r_x = 1 \quad (15)$$

Similarly, the multi-scale correlation principle can be applied on other types of transition curves not discussed here (e.g. the sinusoidal, the sine half-wavelength diminishing straight curve, the Bloss transition, the cubic transition and the bi-quadratic transition curve) [Bloss et al, 1979; Meek and Walton 1989; Meek and Walton, 1992]. Once a prototype of each curve type is stored in a library file it becomes easy to alter its nominal design parameters by issuing the appropriate scale transformation with the minimum computational effort.

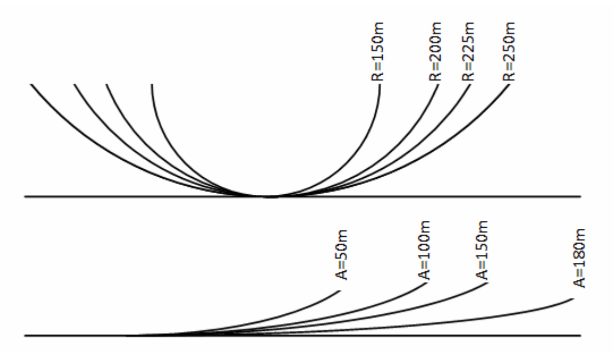

Figure 1. A set of reference circles and clothoid transitions of varying design parameters produced using multi-scale correlation analysis

3. THE DIRECT CURVE MATCHING ALGORITHM

Once a pool of reference curves has generated, curve matching between sample (observed) and reference data is accomplished using the DCM technique. The criterion for the selection of the best matching reference curve is based upon the residual values obtained from comparing the sample data against a multiple of reference curves. In fact, the reference curve with minimum RMS value is said to match best the sample data, and consequently its parameter value is assigned to the sample data set. In mathematical terms, the RMS value of the residuals between the reference y-coordinates Y_R and the sample y-coordinates Y_S , *Er* is given by:

$$Er = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_{Ri} - Y_{Si})^2} \quad (16)$$

Also, the best fitting reference curve to the sample data exhibits a minimum RMS value given by:

$$Er(r_y) = \min \tag{17}$$

Note that in Eq.17 only the r_y scale factor appears. This is because r_x is set either equal to r_y or equal to unity depending on the type of transformation used.

Eq. 17 can be solved for r_y using well known minimization algorithms from numerical analysis. The specific method applied in this study is described as follows:

- Firstly, the *Er* parameter is calculated from fitting a straight line to the sample data. This error value is named *Ers*. As a straight line corresponds to an infinitely large radius the corresponding curvature value (*Ps*) equals zero.
- Then, the *Er* value is computed for the minimum parameter value allowed for the feature type examined based on road design criteria. Such design criteria limits refer to the minimum arc radius or transition parameter and is named *Pc*. The resulting *Er* value is named *Erc*.
- Consequently, the algorithm enters a loop in which the *Ers* and *Erc* values are calculated repeatedly using curvature values computed from the following equations:

$$Ps^{+} = 0.75 \cdot \frac{Pc^{-} \cdot Ers}{Ers + Erc} \tag{18}$$

$$Pc^{+} = 1.25 \cdot \frac{Pc^{-} \cdot Ers}{Ers + Erc} \tag{19}$$

- At every iteration step the error values *Ers* and *Erc* reduce mutually. The procedure stops when the difference between *Ps* and *Pc* values is under a preset low limit or when the last five results of *Ers* and *Erc* do not differ in their integer parts. In cases where the centerline positions suffer from an increased level of noise this aperture limit may be loosened accordingly.

This algorithm converges fast and provides results of an adequate precision even in the case of excessive noise in the position data. The proportional factors in Eq.18 and 19 are used to adjust the rate of convergence, that will have a positive impact on algorithm stability. More specifically, notwithstanding values 0.75 and 1.25 used in Eq. 18 and 19 were chosen experimentally, they were proved to produce statistically best results.

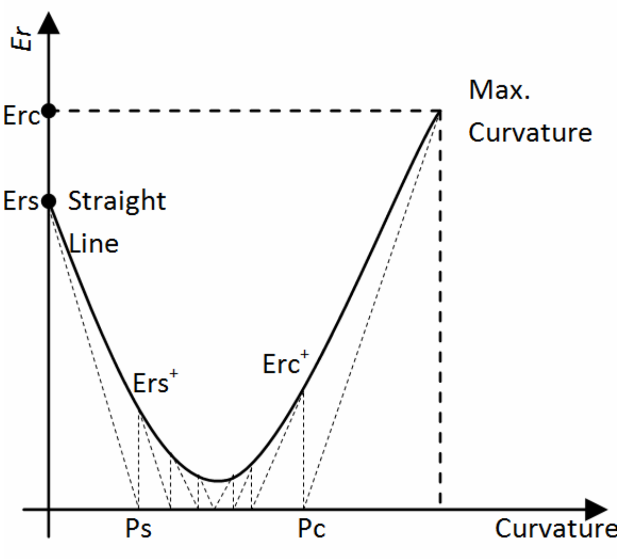

Figure 2. Multi-scale direct curve matching sequence

As an example, Figure 2 depicts the operating principle of the algorithm in the case of an arc circle. In this plot, for practical reasons, the arc curvature is displaced on the x-axis instead of its radius value. The algorithm starts with the calculation of *Erc* and *Ers* errors for the maximum curvature value allowed (lowest circle radius permitted by the design standards) and a straight line (zero curvature). Then, new estimates of *Ps* and *Pc* values are computed using Eq. 18 and 19 and accordingly a pair of scale factors are obtained using Eq. 7 to produce reference circles of a different radius. Consequently, the updated reference arc circles are correlated with the sample data and a new pair of *Erc* and *Ers* values is computed using Eq. 16. The process is repeated until *Ps* and *Pc* values coincide and converge to a minimum *Er* value.

4. THE CENTRELINE MODELING ALGORITHM

In this study, the proposed centerline geometry extraction algorithm integrates multi-scale correlation analysis and the DCM technique. As a first step, the navigation data obtained from the mobile mapping system in forward and reverse directions are processed to produce the centerline alignment in the form of individual data points. Various algorithms exist for this purpose; for example see Gikas and Daskalakis (2008). Then, the resultant data file is sampled so that a small portion of the data is loaded for processing at a time. This is achieved using predefined sample length settings.

Data processing is performed sequentially, in a number of computational steps shown in the flowchart diagram of Figure 3. In this process is assumed that sample length it remains constant over time. In fact, a running window of constant width is defined so that at every processing step the last point in the rolling window is replaced with a new point at the front. In order to facilitate comparison between the sample and reference data, each time the sample data set is updated, a rotation and an offset operation is applied to transform sample data from their initial datum to the coordinate system of the reference curve. A subroutine is also used to identify the turn direction (left of right wise) of the curve for later use. At this stage of the process candidate reference curves are produced using prototype curves of predefined design parameters (typically, R=500 m and A=100 m).

Once both sample and reference data sets have been prepared, the multi-scale correlation and the DCM algorithm are being applied in the data. More specifically, this computational stage implements three clones of the same matching technique. The first one returns the radius and the minimum *Er* value that correspond to the best matching arc circle in the sample data. The second and third clones return the clothoid parameter and accompanying *Er* value for a clothoid-tangent and a tangent-clothoid sequences respectively. In conclusion, the algorithm returns three *Er* error values. These correspond to the best matching straight line, arc circle, and clothoid transition. Finally, the sample data are assigned to the design element type with minimum *Er* error value.

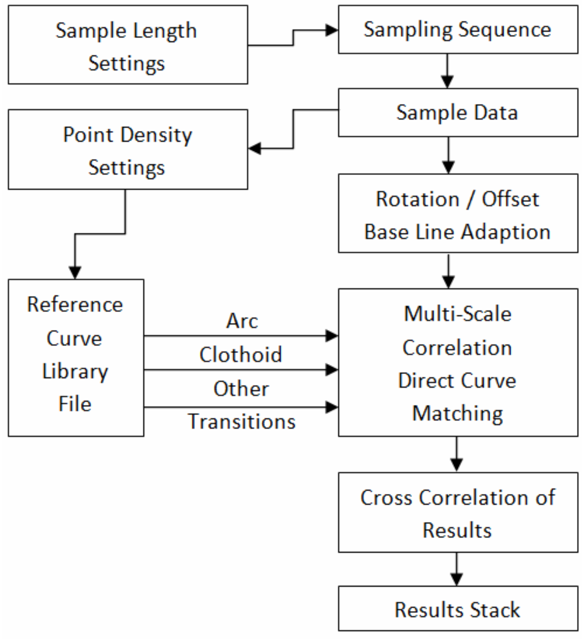

Figure 3. Flowchart of the proposed algorithm

The algorithm is executed repeatedly so that an element type is identified for every subset of data that corresponds to the current location of the running window. Finally, the parameter value of a centerline geometry element is computed from averaging all individual parameter values of the same element type obtained from subsequent loop operations. Notably, for every sample data update the whole correlation set is shifted by a single track point. This attribute is considered important as it facilitates to locate precisely the touch points (characteristic points) between tangent and spiral segments.

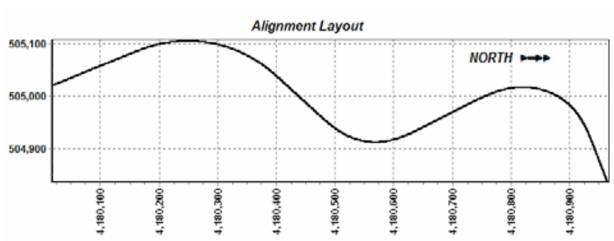

Figure 4. Centreline alignment of the study region. It consists of three sets of spiral-circle-spiral with straight lines in between

5. IMPLEMENTATION AND TESTING OF THE METHOD

5.1 Mobile Mapping Vehicle and Data Description

To verify the correctness and the feasibility of the proposed algorithm software has been written to integrate the complete procedure discussed in Section 4.0. The software was written in the Pascal object oriented programming language. In this paper the results of processing a road alignment of nearly 1300 m long are presented (Figure 4). This section features a multitude of road design elements of varying design parameters. Field data were acquired using a mobile mapping vehicle bearing a high precision GNSS/INS/DMI navigation system, whereas raw data were processed through a Kalman filter algorithm. In addition, the road section was surveyed using conventional geodetic techniques of a higher accuracy. Therefore, a reliability analysis between modeled and "true" geometry would be possible to accomplish.

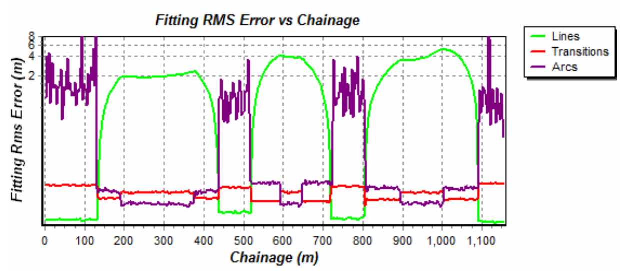

Figure 5. Computed *Er* (RMS) errors for the straight line, circle and clothoid model for the entire alignment.

5.2 Assessment of the Results and Discussion

Data processing assumed a rolling window of a 20-point length size, which corresponds to approximately 1 m travel distance. Similarly, the length and point density of the reference data sets were adjusted accordingly. Figure 5 shows the *Er* (RMS) error values computed for every design element in the alignment. In this plot in order to maintain the lower part of the diagram visible a logarithmic scale is used.

As expected, from this diagram it is evident that the straight line model match best all linear segments in the alignment. In these locations, the clothoid model results in higher error values, whereas the arc circle fit leads to extremely high error values. Clearly, in the linear segment region, neither arc circles nor clothoids can provide a better matching due to constraints applied in both the arc radius and the transition parameter adopted during reference curve generation (R_{max}=5000 m, A_{max}=500 m). Also, the moderate error values observed for the clothoid model (compared to the circle one) are due to the varying curvature values of spiral curves.

Similar conclusions are drawn from modeling the clothoid transition sections. Here, the clothoid model results in minimum *Er* values indicating the best matching between observed and reference data sets. Small discrepancies between the rising and the falling edges in the clothoid-arc-clothoid region they possibly reveal small variations in the vehicle speed resulting in a variable point density between the sample and the reference data sets. Finally, as shown in the same diagram, the arc circle regions are best approximated by the circle model.

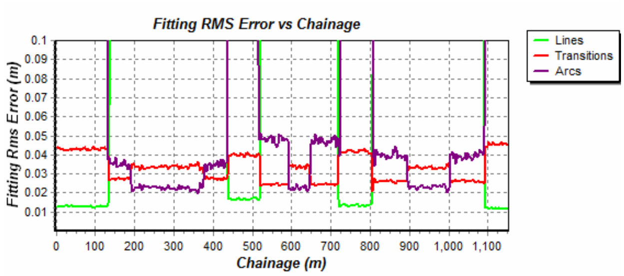

Figure 6. Enlargement of the lower part of Figure 5

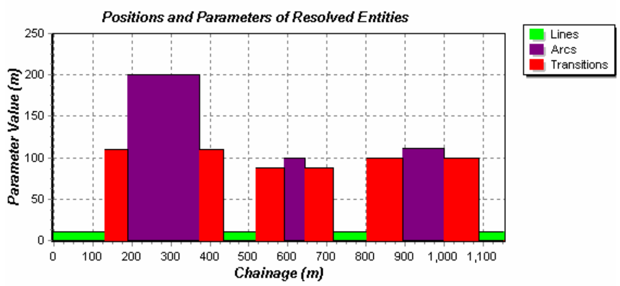

Figure 7. Parameter value versus chainage computed for road alignment examined in this study

Figure 6 presents an enlargement of the lower part of Figure 5. The most important thing to note from this diagram is that the minimum value of the matching error *Er*, is small and remains under 0.03 m. Obviously, such small error values primarily ought to the high quality navigation data. Also, it should be pointed out that every *Er* value relates to a limited data volume (i.e., as much as it corresponds to the sample window size); and therefore, *Er* errors are computed against a very short travel path. Figure 7 shows summary results obtained from processing the centerline data examined in this study. More specifically, this histogram presents parameter value against chainage, while the type of the design element is denoted by color.

	As Built		Measured		% Differences	
Type	Length	R / A	Length	R / A	Length	R / A
Line	129.63	-	131.29	-	1.28	-
Clothoid	60.00	109.55	64.51	111.29	7.51	1.59
Curve	185.36	200.00	185.94	197.86	0.31	1.07
Clothoid	60.00	109.55	63.83	108.61	6.38	0.86
Line	80.21	-	80.71	-	0.62	-
Clothoid	76.00	87.18	78.76	91.52	3.64	4.98
Curve	54.73	100.00	56.16	101.58	2.62	1.58
Clothoid	76.00	87.18	78.94	91.93	3.86	5.45
Line	83.98	-	82.43	-	1.85	-
Clothoid	88.08	99.03	87.84	94.95	0.27	4.12
Curve	107.10	111.33	105.10	111.27	1.86	0.05
Clothoid	88.08	99.03	87.69	98.46	0.44	0.58
Line	145.40	-	146.56	-	0.80	-

Table 1. Comparison of computed versus as-built design parameters

In order to examine the behavior of the algorithm in terms of reliability, a cross-comparison between the computed against as-built alignment is attempted. Table 1 contains summary results for the entire road section. As it can be clearly seen in this table all differences observed stay well under 10% - representing an exceptionally good argument that the model is correct. Also, from the same plot it is evident that the differences in design element length present higher values compared to element parameter values. This observation relates directly to the size of the sample window selected; however, they are still small.

6. CONCLUSIONS

In this paper, a new methodology is presented that deals with the problem of centerline geometry extraction from navigation data in terms of traditional design elements. In contrast with previous methods, the proposed algorithm relies on multi-scale correlation analysis and curve matching techniques; hence, is simple to apply as well as it proved to be computationally efficient. Provided that the algorithm is fed with high quality navigation data, then it can operated in an automatic manner. Also, the algorithm was built to process small subsets of navigational data in a sequential manner, and therefore, can be easily modified to operate in near real time mode. Finally, due to its simplicity, the algorithm is fast enough to cope with high frequency kinematic data. A consideration for further improvement of the method includes the use of a non-constant sample / reference curve length that is expected to fully exploit the spatial characteristics of navigation data.

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